Structural approach to the elementary particle theory

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Abstract

There are two approaches to atomic physics: (1) structural approach and (2) empirical approach (chemistry). The structural approach uses methods of atomic physics and, in particular, quantum mechanics. The structural approach admits one to investigate the structure and arrangement of an atom (nucleus and electronic envelope). The empirical approach uses only experimental methods, in particular, the periodic system of chemical elements. It investigates the properties of chemical elements and their chemical reactions. It can predict new chemical elements and their properties (corresponding quantum numbers), but it cannot investigate the atom arrangement (nucleus and electronic envelope).

In the contemporary theory of elementary particles one has only the empirical approach. It admits one to obtain quantum numbers of elementary particles. It admits one to systematize elementary particles, to investigate their interactions, to predict new elementary particles, but it does not admit one to investigate an arrangement of elementary particles. Structural approach to the elementary particle theory does not exist now. The paper is devoted to development of the structural approach to the elementary particle theory. Being an axiomatic conception, the quantum theory cannot be used in the construction of the structural approach. Considering the quantum motion as statistical description of stochastically moving elementary particle, one succeeded to obtain some elements of the elementary particles arrangement. In particular, it appears, that a relativistic elementary particle generates some force field (κ-field), which is responsible for the pair production. Some properties of the κ-field are investigated in this paper. Stochastic motion of elementary particle can be freely explained by properties of the discrete space-time geometry, which admits one to construct the skeleton conception of elementary particles.

Key words: structural approach; united formalism of dynamics; multivariant geometry; skeleton conception

1 Introduction

There are two different approaches to the elementary particle theory: (1) structural approach and (2) empirical approach. At the structural approach one attempts to investigate an arrangement of elementary particles and their structure. At the empirical approach one distinguishes between the different elementary particles by some "quantum numbers" ascribed to any elementary particle. These "quantum numbers" (parameters) are: mass, electric charge, spin, magnetic moment, baryon charge, isospin and so on. One can classify elementary particles by these parameters an predict new elementary particles on the basis of this classification. However, one cannot connect these parameters with the structure of elementary particles, because in the contemporary theory the structural approach is absent.

What is the structural approach one can understand in the example of the atomic theory, where there are both structural approach and empirical approach. At the structural approach one investigates the atom arrangement: its nucleus and electronic envelope. One uses the quantum mechanics, which admits one to calculate parameters of electronic envelopes of different atoms. At the empirical approach one classifies chemical elements by their properties, generated by parameters of electronic envelopes of their atoms. At the empirical approach one does not interested in structure and arrangement of atoms. At the empirical approach one uses periodical system of chemical elements which has been obtained from experiment. Empirical approach does not permit to investigate the atomic structure. Investigation methods of empirical approach cannot be used at investigation of the atomic structure. The structural (physical) approach is more fundamental, than the empirical (chemical) approach.

In the contemporary theory of elementary particles the structural approach is absent. As a matter of fact the contemporary elementary particle theory is a chemistry (not physics) of elementary particles. Methods of the contemporary theory of elementary particles do not admit to investigate structure (arrangement) of elementary particles. They admits only to ascribe quantum numbers to different elementary particles and distinguish between them by these quantum numbers. The reason of such a situation is a consideration of the quantum laws as fundamental laws of nature, whereas they describe only a mean motion of quantum particles. In the same way the laws of the gas dynamics describe only the mean motion of the gas molecules. Basing on the laws of the gas dynamics, one cannot investigate structure of gas molecules . In this paper the conceptual problems of the microcosm physics will be considered. The structural approach is based on a new conception of elementary particles.

It should note that we distinguish between a conception and a theory. A conception does not coincide with a theory. For instance, the skeleton conception of elementary particles [1] distinguishes from a theory of elementary particles. A conception investigates connections between concepts of a theory. For instance, the skeleton conception of elementary particles investigates the structure of a possible theory of elementary particles. It investigates, why an elementary particle is described by its skeleton (several space-time points), which contains all information on the elementary particle. The skeleton conceptions explains, why dynamic equations are coordinateless algebraic equations and why the dynamic equations a written in terms of the world functions. However, the skeleton conception does not answer the question, which skeleton corresponds to a concrete elementary particle and what is the world function of the real space-time. In other words, the skeleton conception deals with physical principles, but not with concrete elementary particles. The conception cannot be experimentally tested. However, if the world function of the real space-time geometry has been determined, and correspondence between a concrete elementary particle and its skeleton has been established, the skeleton conception turns to the elementary particle theory. The theory of elementary particle (but not a conception) can be tested experimentally.

In other words, it is useless to speak on experimental test of the skeleton conception, because it deals only with physical principles. Discussing properties of a conception, one should discuss only properties of the concepts and logical connection between them, but not to what extent they agree with experimental data. For instance, the statement, that dynamics of deterministic particles is described in terms of Lagrangians is a statement of the particle dynamics conception. It does not state what namely Lagrangian is used for some concrete particle. One obtains a theory of particle dynamics, when it is pointed which Lagrangian describes any particle. The Ptolemaic conception of the planet motion differs from the Newtonian conception of the planet motion, although experimentally both conceptions give the same result for the first six planets of Solar system.

To use the quantum theory in the structural approach, one needs to replace the axiomatic conception of the quantum theory by the model conception. For instance, the wave function is the main object of quantum mechanics. But what is the wave function? From where did it appear? Nobody knows. The wave function is a method of description of any nondissipative continuous medium [2]. The fact that the Schrödinger equation describes a nonrotational flow of some "quantum" fluid was known from the beginning of the quantum mechanics [3, 4]. However, in these cases one started from the Schrödinger equation and quantum principles. One failed to start from hydrodynamics and to conclude quantum description in terms of the world function. To make this, one needs to integrate hydrodynamic equations and to present them in terms of hydrodynamic potentials (Clebsh potentials [5, 6]). Thereafter one can construct the wave function from hydrodynamic potentials and obtain description in terms of the wave function. Generally speaking, the dynamic equation in terms of the wave function is nonlinear. It becomes linear only for nonrotational flow. But linearity of dynamic equations written in terms of the world function is considered as a principle of the quantum mechanics.

But from where does the continuous medium appear at the description of quantum particles? Any quantum particle is stochastic particle, and there are no dynamic equation for description of a single stochastic particle. One can describe only a mean motion of a quantum particle. One needs to consider many independent identical stochastic particles. These particles form a statistical ensemble, which can be considered as a set of stochastic independent particles. The statistical ensemble is a dynamic system of the type of a continuous medium. This statistical ensemble may be considered as a gas of noninteracting stochastic particles. This gas (continuous medium) is described by the wave function. Such a description is convenient, because the dynamic equation is linear in terms of the wave function for nonrotational flows. Such a description explains, from where the wave function appears, and what it means.

Statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ of stochastic particles $\mathcal{S}_{\mathrm{st}}$ is a set of many independent identical stochastic particles $\mathcal{S}_{\mathrm{st}}$. Stochastic particle $\mathcal{S}_{\mathrm{st}}$ is not a dynamical system, and there are no dynamic equations for $\mathcal{S}_{\mathrm{st}}$. However, the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ of stochastic particles $\mathcal{S}_{\mathrm{st}}$ is a dynamic system, and there exist dynamic equations for $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$. The dynamic system $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ is a dynamic system of the type of a continuous medium (fluid). Dynamic equations for $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ describe a mean motion of the stochastic particle $\mathcal{S}_{\mathrm{st}}$. Formally the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ can be considered as a set (not statistical ensemble) of identical deterministic particles $\mathcal{S}_{\mathrm{d}}$ interacting between themselves by means of some force field. If this force field is considered as an attribute of the stochastic particle, then, investigating properties of this force field, one may investigate a structure of the stochastic particle.

For instance, the action for the statistical ensemble of stochastic particles $\mathcal{S}_{\mathrm{st}}$ has the form

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}\left[\mathbf{x},\mathbf{u}\right]=\int\int_{V_{\xi}}\left\{\frac{m}{2}\dot{\mathbf{x}}^{2}+\frac{m}{2}\mathbf{u}^{2}-\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right\}\rho_{1}\left(\boldsymbol{\xi}\right)dtd\boldsymbol{\xi},\qquad\dot{\mathbf{x}}\equiv\frac{d\mathbf{x}}{dt}\tag{1.1}$$

The variable $\mathbf{x}=\mathbf{x}\left(t,\boldsymbol{\xi}\right)$ describes the regular component of the particle motion. The independent variables $\boldsymbol{\xi}=\left\{\xi_1,\xi_2,\xi_3\right\}$ label elements (particles) of the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$. The variable $\mathbf{u}=\mathbf{u}\left(t,\mathbf{x}\right)$ describes the mean value of the stochastic velocity component, $\hbar$ is the quantum constant, $\rho_1\left(\boldsymbol{\xi}\right)$ is a weight function. One may set $\rho_1=1$. The second term in (1.1) describes the kinetic energy of the stochastic velocity component. The third term describes interaction between the stochastic component $\mathbf{u}\left(t,\mathbf{x}\right)$ and the regular component $\dot{\mathbf{x}}\left(t,\boldsymbol{\xi}\right)$. The operator

$$\boldsymbol{\nabla}=\left\{\frac{\partial}{\partial x^{1}},\frac{\partial}{\partial x^{2}},\frac{\partial}{\partial x^{3}}\right\}\tag{1.2}$$

is defined in the space of coordinates $\mathbf{x}$.

Formally the action (1.1) describes a set of deterministic particles $\mathcal{S}_{\mathrm{d}}$, interacting via the force field $\mathbf{u}$. The particles $\mathcal{S}_{\mathrm{d}}$ form a gas (or a fluid), described by the variables $\dot{\mathbf{x}}\left(t,\boldsymbol{\xi}\right)=\mathbf{v}\left(t,\boldsymbol{\xi}\right)$. Here this description is produced in the Lagrange representation. Hydrodynamic description is produced in terms of density ρ and velocity $\mathbf{v}$, where

$$\rho=\rho_1 J, \quad J\equiv\frac{\partial\left(\xi_1,\xi_2,\xi_3\right)}{\partial\left(x^1,x^2,x^3\right)} \tag{1.3}$$

Nonrotational flow of this gas is described by the Schrödinger equation [7].

The dynamic equation for the force field $\mathbf{u}$ is obtained as a result of variation of (1.1) with respect to $\mathbf{u}$. It has the form

$$\mathbf{u}=\mathbf{u}\left(t,\mathbf{x}\right)=-\frac{\hbar}{2m}\boldsymbol{\nabla}\ln\rho \tag{1.4}$$

The vector $\mathbf{u}$ describes the mean value of the stochastic velocity component of the stochastic particle $\mathcal{S}_{\mathrm{st}}$. In the nonrelativistic case the force field $\mathbf{u}$ is determined by its source: the fluid density ρ.

In terms of the wave function the action (1.1) takes the form [7]

$$\mathcal{A}[\psi,\psi^*]=\int\left\{\frac{i\hbar}{2}(\psi^*\partial_0\psi-\partial_0\psi^*\cdot\psi)-\frac{\hbar^2}{2m}\boldsymbol{\nabla}\psi^*\cdot\boldsymbol{\nabla}\psi+\frac{\hbar^2}{8m}\rho\boldsymbol{\nabla}s_\alpha\boldsymbol{\nabla}s_\alpha\right\}\mathrm{d}^4x \tag{1.5}$$

where the wave function $\psi=\left\{\begin{smallmatrix}\psi_1\\ \psi_2\end{smallmatrix}\right\}$ has two complex components.

$$\rho=\psi^*\psi, \qquad s_\alpha=\frac{\psi^*\sigma_\alpha\psi}{\rho}, \qquad \alpha=1,2,3 \tag{1.6}$$

σ_α are 2×2 Pauli matrices

$$\sigma_1=\left(\begin{array}{cc}0&1\\1&0\end{array}\right), \qquad \sigma_2=\left(\begin{array}{cc}0&-i\\i&0\end{array}\right), \qquad \sigma_3=\left(\begin{array}{cc}1&0\\0&-1\end{array}\right), \tag{1.7}$$

Dynamic equation, generated by the action (1.5), has the form

$$i\hbar\partial_0\psi+\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi+\frac{\hbar^2}{8m}\boldsymbol{\nabla}^2 s_\alpha\cdot\left(s_\alpha-2\sigma_\alpha\right)\psi-\frac{\hbar^2}{4m}\frac{\boldsymbol{\nabla}\rho}{\rho}\boldsymbol{\nabla}s_\alpha\sigma_\alpha\psi=0 \tag{1.8}$$

In the case of one-component wave function ψ, when the flow is nonrotational and $\boldsymbol{\nabla}s_\alpha=0$, the dynamic equation has the form of the Schrödinger equation

$$i\hbar\partial_0\psi+\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi=0 \tag{1.9}$$

Thus, the Schrödinger equation is a special case of the dynamic equation, generated by the action (1.1) or (1.5).

There are several interpretations of the Schrödinger particle $\mathcal{S}_{\mathrm{st}}$: (1) quantum interpretation, (2) hydrodynamic interpretation, (3) dynamic interpretation.

At the conventional quantum interpretation the Schrödinger particle $\mathcal{S}_{\mathrm{st}}$ is a quantum particle, whose dynamics is described by the axiomatic Schrödinger equation (1.9). Any questions of the type: why the particle $\mathcal{S}_{\mathrm{st}}$ is quantum and what its parameters are responsible for its quantum behavior, are improper because of axiomatic character of description.

The hydrodynamic interpretation and dynamic interpretation are rather close. According to hydrodynamic interpretation the action (1.1) describes a set of deterministic particles interacting between themselves via the force field $\mathbf{u}$. One cannot consider a single particle, because in this case interaction between particles disappears. The hydrodynamic description does not admit a consideration of a single particle. When the action (1.1) describes a statistical ensemble (not a set of interacting identical deterministic particles), the dynamic interpretation admits one to consider experiments with a single stochastic particle. Experiments with the statistical ensemble can be realized as a set of experiments with identically prepared stochastic particles. Motion of single particles will be different, in general, in different experiments. However, result of the statistical handling of all experiments does not depend on the way of the experiments realization. Experiments with a single particle may be produced simultaneously at the same place or at different places in different time. The statistical averaging gives the same result in all these cases. For instance, the two-slit experiment can be produced with many electrons simultaneously, or with a single electron many times. Result of the statistical averaging will be the same in all cases. It shows, that the action (1.1) describes a statistical ensemble of stochastic particles, but not a gas of interacting deterministic particles.

Thus, the dynamic interpretations, when the action (1.1) describes the statistical ensemble of stochastic particles is the most true interpretation, which does not close the door for investigation of the stochastic behavior of quantum particles. The reason of such a stochastic behavior may be an interaction of a particle with the medium (vacuum, or ether), where the particle moves. The influence of the medium remains the same in all single experiments.

2 Relativistic stochastic particle

The pair production phenomenon takes place only for relativistic quantum particles. It is absent for classical particles. What is the reason of the pair production? Can it be described dynamically? A dynamic description is impossible in the framework of conventional axiomatic quantum theory. However, it is possible at the description of $\mathcal{S}_{\text{st}}$ in terms of the statistical ensemble. In the case of relativistic stochastic particle $\mathcal{S}_{\text{st}}$ the force field has its own degrees of freedom. It can escape from the source and travel in the space-time. In the relativistic case one obtains the action

$$\mathcal{A}[x,\kappa] = \int \left\{ -mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k} - \frac{e}{c}A_k\dot{x}^k \right\} d^4\xi, \quad d^4\xi = d\xi_0 d\boldsymbol{\xi} \tag{2.1}$$

$$K = \sqrt{1+\lambda^2\left(\kappa_l\kappa^l + \partial_l\kappa^l\right)}, \qquad \lambda = \frac{\hbar}{mc}, \quad \tau = \boldsymbol{\xi}_0 \tag{2.2}$$

Here $x = \{x^i(\xi_0, \boldsymbol{\xi})\}$, $i = 0,1,2,3$ are dependent variables, describing regular component of the particle motion. The variables $\xi = \{\xi_0, \boldsymbol{\xi}\} = \{\xi_k\}$, $k = 0,1,2,3$ are independent variables, labelling the particles of the statistical ensemble, and $\dot{x}^i \equiv dx^i/d\xi_0$. The quantities $\kappa^l = \left\{\kappa^l(x)\right\}$, $l = 0,1,2,3$ are dependent variables, describing stochastic component of the particle motion, $A_k = \{A_k(x)\}$, $k = 0,1,2,3$

is the potential of electromagnetic field. We shall refer to the dynamic system, described by the action (2.1), (2.2) as $\mathcal{S}_{\mathrm{KG}}$, because irrotational flow of $\mathcal{S}_{\mathrm{KG}}$ is described by the Klein-Gordon equation [8]. We present here this transformation to the Klein-Gordon form. Here and farther a summation is produced over repeated Latin indices $(0 \div 3)$ and over Greek indices $(1 \div 3)$. We present here transformation of (2.1), (2.2) to the Klein-Gordon form.

Dynamic equations generated by the action (2.1), (2.2) are equations of the hydrodynamical type. To present these equations in terms of the wave function, one needs to integrate them in general form. The problem of general integration of four hydrodynamic Euler equations

$$\partial_0\rho + \boldsymbol{\nabla}(\rho\mathbf{v}) = 0 \tag{2.3}$$

$$\partial_0\mathbf{v}+(\mathbf{v}\boldsymbol{\nabla})\mathbf{v} = -\frac{1}{\rho}\boldsymbol{\nabla}p, \qquad p = p(\rho, \boldsymbol{\nabla}\rho) \tag{2.4}$$

seems to be hopeless. It is really so, if the Euler system (2.3), (2.4) is considered to be a complete system of dynamic equations. In fact, the Euler equations (2.3), (2.4) do not form a complete system of dynamic equations, because it does not describe motion of fluid particles along their trajectories. To obtain the complete system of dynamic equations, we should add to the Euler system so called Lin constraints [9]

$$\partial_0\boldsymbol{\xi} + (\mathbf{v}\boldsymbol{\nabla})\boldsymbol{\xi} = 0 \tag{2.5}$$

where $\boldsymbol{\xi} = \boldsymbol{\xi}(t, \mathbf{x}) = \{\xi_1, \xi_2, \xi_3\}$ are three independent integrals of dynamic equations

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(t, \mathbf{x}),$$

describing motion of fluid particles in the give velocity field.

Seven equations (2.3) – (2.5) form the complete system of dynamic equations, whereas four Euler equations (2.3), (2.4) form only a closed subsystem of the complete system of dynamic equations. The wave function is expressed via hydrodynamic potentials $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, which are known also as Clebsch potentials [5, 6]. In general case of arbitrary fluid flow in three-dimensional space the complex wave function ψ has two complex components ψ_1, ψ_2 (or four independent real components)

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \sqrt{\rho}e^{i\varphi}u_1(\boldsymbol{\xi}) \\ \sqrt{\rho}e^{i\varphi}u_2(\boldsymbol{\xi}) \end{pmatrix}, \qquad |u_1|^2 + |u_2|^2 = 1 \tag{2.6}$$

It is impossible to obtain general solution of the Euler system (2.3), (2.4), but one can partially integrate the complete system (2.3) – (2.5), reducing its order to four dynamic equations for the wave function (2.6). Practically it means that one integrates dynamic equations (2.5), where the function $\mathbf{v}(t, \mathbf{x})$ is determined implicitly by equations (2.3), (2.4). Such an integration and reduction of the order of the complete system of dynamic equations appear to be possible, because the system (2.3) – (2.5) has the symmetry group, connected with transformations of the Clebsch potentials

$$\xi_\alpha \rightarrow \tilde{\xi}_\alpha = \tilde{\xi}_\alpha(\boldsymbol{\xi}), \qquad \alpha = 1,2,3, \qquad \frac{\partial\left(\tilde{\xi}_1, \tilde{\xi}_2, \tilde{\xi}_3\right)}{\partial\left(\xi_1, \xi_2, \xi_3\right)} \neq 0 \tag{2.7}$$

3 Transformation of the action to description in terms of the wave function

Let us consider variables $\xi = \xi(x)$ in (2.1) as dependent variables and variables x as independent variables. Let the Jacobian

$$J = \frac{\partial\left(\xi_0, \xi_1, \xi_2, \xi_3\right)}{\partial\left(x^0, x^1, x^2, x^3\right)} = \det\left|\left|\xi_{i,k}\right|\right|, \qquad \xi_{i,k} \equiv \partial_k \xi_i, \qquad i,k = 0,1,2,3 \tag{3.1}$$

be considered to be a multilinear function of $\xi_{i,k}$. Then

$$d^4\xi = J d^4 x, \qquad \dot{x}^i \equiv \frac{dx^i}{d\xi_0} \equiv \frac{\partial\left(x^i, \xi_1, \xi_2, \xi_3\right)}{\partial\left(\xi_0, \xi_1, \xi_2, \xi_3\right)} = J^{-1}\frac{\partial J}{\partial \xi_{0,i}} \tag{3.2}$$

After transformation to dependent variables ξ the action (2.1) takes the form

$$\mathcal{A}[\xi, \kappa] = \int \left\{-mcK\sqrt{g_{ik}\frac{\partial J}{\partial \xi_{0,i}}\frac{\partial J}{\partial \xi_{0,k}}} - \frac{e}{c}A_k\frac{\partial J}{\partial \xi_{0,k}}\right\} d^4x, \tag{3.3}$$

$$K = \sqrt{1 + \lambda^2\left(\kappa_l \kappa^l + \partial_l \kappa^l\right)}, \qquad \lambda = \frac{\hbar}{mc}, \tag{3.4}$$

Let us introduce new variables

$$j^k = \frac{\partial J}{\partial \xi_{0,k}}, \qquad k = 0,1,2,3 \tag{3.5}$$

by means of Lagrange multipliers p_k

$$\mathcal{A}[\xi, \kappa, j, p] = \int \left\{-mcK\sqrt{g_{ik}j^i j^k} - \frac{e}{c}A_k j^k + p_k\left(\frac{\partial J}{\partial \xi_{0,k}} - j^k\right)\right\} d^4x, \tag{3.6}$$

Variation with respect to ξ_i gives

$$\frac{\delta \mathcal{A}}{\delta \xi_i} = -\partial_l\left(p_k \frac{\partial^2 J}{\partial \xi_{0,k}\partial \xi_{i,l}}\right) = 0, \qquad i = 0,1,2,3 \tag{3.7}$$

Using identities

$$\frac{\partial^2 J}{\partial \xi_{0,k}\partial \xi_{i,l}} \equiv J^{-1}\left(\frac{\partial J}{\partial \xi_{0,k}}\frac{\partial J}{\partial \xi_{i,l}} - \frac{\partial J}{\partial \xi_{0,l}}\frac{\partial J}{\partial \xi_{i,k}}\right) \tag{3.8}$$

$$\frac{\partial J}{\partial \xi_{i,l}}\xi_{k,l} \equiv J\delta_k^i, \qquad \partial_l \frac{\partial^2 J}{\partial \xi_{0,k}\partial \xi_{i,l}} \equiv 0 \tag{3.9}$$

one can test by direct substitution that the general solution of linear equations (3.7) has the form

$$p_k = b_0 \left(\partial_k \varphi + g^{\alpha}(\boldsymbol{\xi}) \partial_k \xi_\alpha\right), \qquad k = 0,1,2,3 \tag{3.10}$$

where $b_0 \neq 0$ is a constant, $g^{\alpha}(\boldsymbol{\xi})$, $\alpha = 1,2,3$ are arbitrary functions of $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, and φ is the dynamic variable ξ_0, which stops to be fictitious. Let us substitute (3.10) in (3.6). The term of the form $\partial J/\partial \xi_{0,k}\partial_k \varphi$ is reduced to Jacobian and does not contribute to dynamic equation. The terms of the form $\xi_{\alpha,k}\partial J/\partial \xi_{0,k}$ vanish due to identities (3.9). We obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa, j] = \int \left\{-mcK\sqrt{g_{ik}j^i j^k} - j^k \pi_k\right\} d^4x, \tag{3.11}$$

where quantities π_k are determined by the relations

$$\pi_k = b_0 \left(\partial_k \varphi + g^{\alpha}(\boldsymbol{\xi}) \partial_k \xi_\alpha\right) + \frac{e}{c}A_k, \qquad k = 0,1,2,3 \tag{3.12}$$

Integration of (3.7) in the form (3.10) is that integration which admits to introduce a wave function. Note that coefficients in the system of equations (3.7) at derivatives of p_k are constructed of minors of the Jacobian (3.1). It is the circumstance that admits to produce a formal general integration.

Variation of (3.11) with respect to κ^l gives

$$\frac{\delta \mathcal{A}}{\delta \kappa^l} = -\frac{\lambda^2 mc\sqrt{g_{ik}j^i j^k}}{K}\kappa_l + \partial_l \frac{\lambda^2 mc\sqrt{g_{ik}j^i j^k}}{2K} = 0, \quad \lambda = \frac{\hbar}{mc} \tag{3.13}$$

It can be written in the form

$$\kappa_l = \partial_l \kappa = \frac{1}{2}\partial_l \ln \rho, \qquad e^{2\kappa} = \frac{\rho}{\rho_0} \equiv \frac{\sqrt{j_s j^s}}{\rho_0 K}, \tag{3.14}$$

where ρ_0 =const is the integration constant. Substituting (3.4) in (3.14), we obtain dynamic equation for κ

$$\hbar^2 \left(\partial_l \kappa \cdot \partial^l \kappa + \partial_l \partial^l \kappa\right) = m^2 c^2 \frac{e^{-4\kappa} j_s j^s}{\rho_0^2} - m^2 c^2 \tag{3.15}$$

Variation of (3.11) with respect to j^k gives

$$\pi_k = -\frac{mcKj_k}{\sqrt{g_{ls}j^l j^s}} \tag{3.16}$$

or

$$\pi_k g^{kl} \pi_l = m^2 c^2 K^2 \tag{3.17}$$

Substituting $\sqrt{j_s j^s}/K$ from the second equation (3.14) in (3.16), we obtain

$$j_k = -\frac{\rho_0}{mc} e^{2\kappa} \pi_k, \tag{3.18}$$

Now we eliminate the variables j^k from the action (3.11), using relation (3.18) and (3.14). We obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa] = \int \rho_0 e^{2\kappa} \left\{-m^2 c^2 K^2 + \pi^k \pi_k\right\} d^4x, \tag{3.19}$$

where π_k is determined by the relation (3.12). Using expression (2.2) for K, the first term of the action (3.19) can be transformed as follows.

$$\begin{aligned} -m^2c^2e^{2\kappa}K^2 &= -m^2c^2e^{2\kappa}\left(1+\lambda^2\left(\partial_l\kappa\partial^l\kappa+\partial_l\partial^l\kappa\right)\right) \\ &= -m^2c^2e^{2\kappa}+\hbar^2e^{2\kappa}\partial_l\kappa\partial^l\kappa-\frac{\hbar^2}{2}\partial_l\partial^l e^{2\kappa} \end{aligned}$$

Let us take into account that the last term has the form of divergence. It does not contribute to dynamic equations and can be omitted. Omitting this term, we obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa] = \int \rho_0 e^{2\kappa} \left\{-m^2 c^2 + \hbar^2 \partial_l\kappa\partial^l\kappa + \pi^k \pi_k\right\} d^4x, \tag{3.20}$$

Here π_k is defined by the relation (3.12), where the integration constant b_0 is chosen in the form $b_0 = \hbar$

$$\pi_k = \hbar\left(\partial_k\varphi + g^\alpha(\boldsymbol{\xi})\,\partial_k\xi_\alpha\right) + \frac{e}{c}A_k, \qquad k = 0,1,2,3 \tag{3.21}$$

Instead of dynamic variables $\varphi, \boldsymbol{\xi}, \kappa$ we introduce n-component complex function

$$\psi = \{\psi_\alpha\} = \left\{\sqrt{\rho}e^{i\varphi}u_\alpha(\boldsymbol{\xi})\right\} = \left\{\sqrt{\rho_0}e^{\kappa+i\varphi}u_\alpha(\boldsymbol{\xi})\right\}, \qquad \alpha = 1,2,...n \tag{3.22}$$

Here u_α are functions of only $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, having the following properties

$$\sum_{\alpha=1}^{\alpha=n} u_\alpha^* u_\alpha = 1, \qquad -\frac{i}{2}\sum_{\alpha=1}^{\alpha=n}\left(u_\alpha^*\frac{\partial u_\alpha}{\partial\xi_\beta} - \frac{\partial u_\alpha^*}{\partial\xi_\beta}u_\alpha\right) = g^\beta(\boldsymbol{\xi}) \tag{3.23}$$

where $(^*)$ denotes the complex conjugation. The number n of components of the wave function ψ depends on the functions $g^\beta(\boldsymbol{\xi})$. The number n is chosen in such a way, that equations (3.23) have a solution. Then we obtain

$$\psi^*\psi \equiv \sum_{\alpha=1}^{\alpha=n}\psi_\alpha^*\psi_\alpha = \rho = \rho_0 e^{2\kappa}, \qquad \partial_l\kappa = \frac{\partial_l(\psi^*\psi)}{2\psi^*\psi} \tag{3.24}$$

$$\pi_k = -\frac{i\hbar\left(\psi^*\partial_k\psi - \partial_k\psi^*\cdot\psi\right)}{2\psi^*\psi} + \frac{e}{c}A_k, \qquad k = 0,1,2,3 \tag{3.25}$$

Substituting relations (3.24), (3.25) in (3.20), we obtain the action, written in terms of the wave function ψ

$$\begin{aligned}\mathcal{A}[\psi,\psi^*] &= \int\left\{\left[\frac{i\hbar\left(\psi^*\partial_k\psi-\partial_k\psi^*\cdot\psi\right)}{2\psi^*\psi}-\frac{e}{c}A_k\right]\left[\frac{i\hbar\left(\psi^*\partial^k\psi-\partial^k\psi^*\cdot\psi\right)}{2\psi^*\psi}-\frac{e}{c}A^k\right]\right.\\ &\left.+\hbar^2\frac{\partial_l\left(\psi^*\psi\right)\partial^l\left(\psi^*\psi\right)}{4\left(\psi^*\psi\right)^2}-m^2c^2\right\}\psi^*\psi d^4x \qquad (3.26)\end{aligned}$$

Let us use the identity

$$\begin{aligned}&\frac{\left(\psi^*\partial_l\psi-\partial_l\psi^*\cdot\psi\right)\left(\psi^*\partial^l\psi-\partial^l\psi^*\cdot\psi\right)}{4\psi^*\psi}+\partial_l\psi^*\partial^l\psi\\ \equiv\ &\frac{\partial_l\left(\psi^*\psi\right)\partial^l\left(\psi^*\psi\right)}{4\psi^*\psi}+\frac{g^{ls}}{2}\psi^*\psi\sum_{\alpha,\beta=1}^{\alpha,\beta=n}Q^*_{\alpha\beta,l}Q_{\alpha\beta,s} \qquad (3.27)\end{aligned}$$

where

$$Q_{\alpha\beta,l}=\frac{1}{\psi^*\psi}\begin{vmatrix}\psi_\alpha & \psi_\beta\\ \partial_l\psi_\alpha & \partial_l\psi_\beta\end{vmatrix},\qquad Q^*_{\alpha\beta,l}=\frac{1}{\psi^*\psi}\begin{vmatrix}\psi^*_\alpha & \psi^*_\beta\\ \partial_l\psi^*_\alpha & \partial_l\psi^*_\beta\end{vmatrix} \qquad (3.28)$$

Then we obtain

$$\begin{aligned}\mathcal{A}[\psi,\psi^*] &= \int\left\{\left(i\hbar\partial_k+\frac{e}{c}A_k\right)\psi^*\left(-i\hbar\partial^k+\frac{e}{c}A^k\right)\psi-m^2c^2\psi^*\psi\right.\\ &\left.+\frac{\hbar^2}{2}\sum_{\alpha,\beta=1}^{\alpha,\beta=n}g^{ls}Q_{\alpha\beta,l}Q^*_{\alpha\beta,s}\psi^*\psi\right\}d^4x \qquad (3.29)\end{aligned}$$

Let us consider the case of irrotational flow, when $g^\alpha(\boldsymbol{\xi})=0$ and the function ψ has only one component. It follows from (3.28), that $Q_{\alpha\beta,l}=0$. Then we obtain instead of (3.29)

$$\mathcal{A}[\psi,\psi^*]=\int\left\{\left(i\hbar\partial_k+\frac{e}{c}A_k\right)\psi^*\left(-i\hbar\partial^k+\frac{e}{c}A^k\right)\psi-m^2c^2\psi^*\psi\right\}d^4x \qquad (3.30)$$

Variation of the action (3.30) with respect to ψ^* generates the Klein-Gordon equation

$$\left(-i\hbar\partial_k+\frac{e}{c}A_k\right)\left(-i\hbar\partial^k+\frac{e}{c}A^k\right)\psi-m^2c^2\psi=0 \qquad (3.31)$$

Thus, description in terms of the Klein-Gordon equation is a special case of the stochastic particles description by means of the action (2.1), (2.2).

In the case, when the fluid flow is rotational, and the wave function ψ is two-component, the identity (3.27) takes the form

$$\begin{aligned}&\frac{\left(\psi^*\partial_l\psi-\partial_l\psi^*\cdot\psi\right)\left(\psi^*\partial^l\psi-\partial^l\psi^*\cdot\psi\right)}{4\rho}-\frac{\left(\partial_l\rho\right)\left(\partial^l\rho\right)}{4\rho}\\ \equiv\ &-\partial_l\psi^*\partial^l\psi+\frac{1}{4}\left(\partial_l s_\alpha\right)\left(\partial^l s_\alpha\right)\rho \qquad (3.32)\end{aligned}$$

where 3-vector $\mathbf{s} = \{s_1, s_2, s_3,\}$ is defined by the relation

$$\rho = \psi^*\psi, \qquad s_\alpha = \frac{\psi^*\sigma_\alpha\psi}{\rho}, \qquad \alpha = 1,2,3 \tag{3.33}$$

$$\psi = \binom{\psi_1}{\psi_2}, \qquad \psi^* = (\psi_1^*, \psi_2^*), \tag{3.34}$$

and Pauli matrices $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ have the form

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{3.35}$$

Note that 3-vectors $\mathbf{s}$ and $\boldsymbol{\sigma}$ are vectors in the space V_ξ of the Clebsch potentials $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$. They transform as vectors at the transformations (2.7)

In general, transformations of Clebsch potentials $\boldsymbol{\xi}$ and those of coordinates $\mathbf{x}$ are independent. However, the action (3.26) does not contain any reference to the Clebsch potentials $\boldsymbol{\xi}$ and transformations (2.7) of $\boldsymbol{\xi}$. If we consider only linear transformations of space coordinates $\mathbf{x}$

$$x^\alpha \rightarrow \tilde{x}^\alpha = b^\alpha + \omega^\alpha_{.\beta} x^\beta, \qquad \alpha = 1,2,3 \tag{3.36}$$

nothing prevents from accompanying any transformation (3.36) with the similar transformation

$$\xi_\alpha \rightarrow \tilde{\xi}_\alpha = b^\alpha + \omega^\alpha_{.\beta} \xi_\beta, \qquad \alpha = 1,2,3 \tag{3.37}$$

of Clebsch potentials $\boldsymbol{\xi}$. The formulas for linear transformation of vectors and spinors in V_x do not contain the coordinates $\mathbf{x}$ explicitly, and one can consider vectors and spinors in V_ξ as vectors and spinors in V_x, provided we consider linear transformations (3.36), (3.37) always together.

Using identity (3.32), we obtain from (3.26)

$$\mathcal{A}[\psi, \psi^*] = \int \left\{ \left(i\hbar\partial_k + \frac{e}{c}A_k \right) \psi^* \left(-i\hbar\partial^k + \frac{e}{c}A^k \right) \psi - m^2c^2\rho - \frac{\hbar^2}{4} (\partial_l s_\alpha) \left(\partial^l s_\alpha \right) \rho \right\} d^4x \tag{3.38}$$

Dynamic equation, generated by the action (3.38), has the form

$$\begin{aligned} &\left(-i\hbar\partial_k + \frac{e}{c}A_k \right) \left(-i\hbar\partial^k + \frac{e}{c}A^k \right) \psi - \left(m^2c^2 + \frac{\hbar^2}{4} (\partial_l s_\alpha) \left(\partial^l s_\alpha \right) \right) \psi \\ = \quad & -\hbar^2 \frac{\partial_l \left(\rho \partial^l s_\alpha \right)}{2\rho} (\sigma_\alpha - s_\alpha) \psi \end{aligned} \tag{3.39}$$

The gradient of the unit 3-vector $\mathbf{s} = \{s_1, s_2, s_3\}$ describes rotational component of the fluid flow. If $\mathbf{s} = \text{const}$, the dynamic equation (3.39) turns to the conventional Klein-Gordon equation (3.31). Curl of the vector field π_k, determined by the relation (3.25), is expressed only via derivatives of the unit 3-vector $\mathbf{s}$.

To show this, let us represent the wave function (3.22) in the form

$$\psi = \sqrt{\rho} e^{i\varphi} (\mathbf{n}\boldsymbol{\sigma}) \chi, \qquad \psi^* = \sqrt{\rho} e^{-i\varphi} \chi^* (\boldsymbol{\sigma}\mathbf{n}), \qquad \mathbf{n}^2 = 1, \qquad \chi^*\chi = 1 \tag{3.40}$$

where $\mathbf{n} = \{n_1, n_2, n_3\}$ is some unit 3-vector, $\chi = \binom{\chi_1}{\chi_2}$, $\chi^* = (\chi_1^*, \chi_2^*)$ are constant two-component quantities, and $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ are Pauli matrices (3.35). The unit vector $\mathbf{s}$ and the unit vector $\mathbf{n}$ are connected by means of the relations

$$\mathbf{s} = 2\mathbf{n}\,(\mathbf{nz}) - \mathbf{z}, \qquad \mathbf{n} = \frac{\mathbf{s}+\mathbf{z}}{\sqrt{2\,(1+(\mathbf{sz}))}} \tag{3.41}$$

where $\mathbf{z}$ is a constant unit vector defined by the relation

$$\mathbf{z} = \chi^* \boldsymbol{\sigma} \chi, \qquad \mathbf{z}^2 = \chi^*\chi = 1 \tag{3.42}$$

All 3-vectors $\mathbf{n}$, $\mathbf{s}$, $\mathbf{z}$ are vectors in V_ξ. Let us substitute the relation (3.40) into expression $\partial_l \pi_k - \partial_k \pi_l$ for the curl of the vector field π_k defined by the relation (3.25). Then gradually reducing powers of σ by means of the identity

$$\sigma_\alpha \sigma_\beta \equiv \delta_{\alpha\beta} + i\varepsilon_{\alpha\beta\gamma}\sigma_\gamma, \qquad \alpha, \beta = 1,2,3 \tag{3.43}$$

where $\varepsilon_{\alpha\beta\gamma}$ is the Levi-Chivita pseudotensor ($\varepsilon_{123} = 1$), we obtain after calculations

$$\begin{aligned} \pi_k &= -\frac{i\hbar\,(\psi^*\partial_k\psi - \partial_k\psi^*\cdot\psi)}{2\psi^*\psi} + \frac{e}{c}A_k \\ &= \hbar\,(\partial_k\varphi + \varepsilon_{\alpha\beta\gamma} n_\alpha \partial_k n_\beta z_\gamma) + \frac{e}{c}A_k \qquad k = 0,1,2,3 \end{aligned} \tag{3.44}$$

$$\partial_k \pi_l - \partial_l \pi_k = -4\hbar\,[\partial_k \mathbf{n} \times \partial_l \mathbf{n}]\,\mathbf{z} + \frac{e}{c}\,(\partial_k A_l - \partial_l A_k)\,, \qquad k,l = 0,1,2,3 \tag{3.45}$$

The relation (3.45) may be expressed also via the 3-vector $\mathbf{s}$, provided we use the formulae (3.41).

Note that the two-component form of the wave function can describe irrotational flow. For instance, if $\psi = \binom{\psi_1}{\psi_1}$, $s_1 = 1$, $s_2 = s_3 = 0$, the dynamic equation (3.39) reduces to the form (3.31), and curl of π_k, defined by (3.45) reduces to

$$\partial_k \pi_l - \partial_l \pi_k = \frac{e}{c}\,(\partial_k A_l - \partial_l A_k)\,, \qquad k,l = 0,1,2,3 \tag{3.46}$$

4 κ-field is responsible for pair production

The nonrelativistic field $\mathbf{u}$ in the action (1.1) is an internal field of the nonrelativistic particle. It can act only on the motion of the nonrelativistic particle, making it stochastic. According to the action (2.1), (2.2) the κ-field looks also as an internal field of the particle. It seems that it may act only on the motion of the particle, and it cannot act on motion of other particles. However, it is not so. The κ-field (a relativistic version of nonrelativistic field $\mathbf{u}$) can produce pairs. In other words, the κ-field can turn the particle world line in the time direction. Formally, in such an action κ-field acts as an internal field of the particle. But such a turn of the world line is possible only, if the κ-field is a given external field. Let us illustrate this in the example [10], when

$$K=\sqrt{1+\lambda^2\left(\kappa_l\kappa^l+\partial_l\kappa^l\right)}=\sqrt{1+f\left(x\right)} \tag{4.1}$$

where $f\left(x\right)$ is some given function of coordinates x. The action (2.1), (2.2) takes the form

$$\mathcal{A}\left[q\right]=\int L\left(q,\dot{q}\right)d\tau,\qquad L=-\sqrt{m^2c^2\left(1+f\left(q\right)\right)g_{ik}\dot{q}^i\dot{q}^k}-\frac{e}{c}A_k\dot{q}^k \tag{4.2}$$

where relations $x^i=q^i\left(\tau\right),\ i=0,1,2,3$ describe the world line of the particle, and $\dot{q}^k\equiv dq^i/d\tau$. The quantities $A_k=A_k\left(q\right),\ k=0,1,2,3$ are given electromagnetic potentials, and $f=f\left(q\right)$ is some given field, replacing the particle mass m by the effective particle mass $m_{\mathrm{eff}}=m\sqrt{\left(1+f\left(q\right)\right)}$. The canonical momentum p_k is defined by the relation

$$p_k=\frac{\partial L}{\partial\dot{q}^k}=-\frac{mcKg_{ki}\dot{q}^i}{\sqrt{g_{ls}\dot{q}^l\dot{q}^s}}-\frac{e}{c}A_k,\qquad K=\sqrt{\left(1+f\left(q\right)\right)} \tag{4.3}$$

Dynamic equations have the form

$$\frac{dp_k}{d\tau}=-mc\sqrt{g_{ik}\dot{q}^i\dot{q}^k}\frac{\partial K}{\partial q^k}-\frac{e}{c}\frac{\partial A_i}{\partial q^k}\dot{q}^i \tag{4.4}$$

One can see from (4.3), that the vector

$$\dot{q}_k=\sqrt{\frac{g_{ls}\dot{q}^l\dot{q}^s}{1+f\left(q\right)}}\frac{\left(p_k+\frac{e}{c}A_k\right)}{mc} \tag{4.5}$$

becomes to be spacelike $\left(g_{ls}\dot{q}^l\dot{q}^s<0\right)$, if $f\left(q\right)<-1$, because only in this case the expression under radical in (4.5) is real.

The Hamilton-Jacobi equation for the action (4.2) has the form

$$g^{ik}\left(\frac{\partial S}{\partial q^i}+\frac{e}{c}A_i\right)\left(\frac{\partial S}{\partial q^k}+\frac{e}{c}A_k\right)=m^2c^2\left(1+f\left(q\right)\right) \tag{4.6}$$

Let us consider solution of the Hamilton-Jacobi equation in the space-time, where $A_i=0$, and $f=f\left(t\right)$ is a function of only time t. In this case the full integral $S\left(t,\mathbf{x},\mathbf{p}\right)$ of equation (4.6) has the form

$$S\left(t,\mathbf{x},\mathbf{p}\right)=\mathbf{px}+\int\limits_0^t c\sqrt{m^2c^2\left(1+f\left(t\right)\right)+\mathbf{p}^2}dt+C,\qquad \mathbf{p},C=\mathrm{const} \tag{4.7}$$

where $\mathbf{p}=\left\{p_1,p_2,p_3\right\}$ are parameters. The equation of the world line is defined by the equation

$$\frac{\partial S\left(t,\mathbf{x},\mathbf{p}\right)}{\partial p_\alpha}=x^\alpha-x_0^\alpha,\quad x_0^\alpha=\mathrm{const},\quad \alpha=1,2,3 \tag{4.8}$$

Substituting (4.7) in (4.8) and setting $p_2 = p_3 = 0$, one obtains

$$x^1 - x_0^1 + \int\limits_0^t \frac{p_1 c dt}{\sqrt{m^2c^2\left(1 + f\left(t\right)\right) + p_1^2}} = 0, \qquad x_0^1 = \text{const} \tag{4.9}$$

$$x^\alpha = x_0^\alpha = \text{const}, \quad \alpha = 2, 3 \tag{4.10}$$

Let for example

$$f\left(t\right) = \begin{cases} 0 & \text{if} \quad t < 0 \\ -\frac{V^2}{m^2c^4t_0^2} t\left(t - t_0\right) & \text{if} \quad 0 < t < t_0 \\ 0 & \text{if} \quad t_0 < t \end{cases}, \qquad t_0, V = \text{const} \tag{4.11}$$

The world line (4.9) takes the form

$$x^1 = \begin{cases} x_0^1 - \frac{p_1c^2}{E} t & \text{if} \quad t < 0 \\ x_0^1 - \int\limits_0^t \frac{p_1 c dt}{\sqrt{E^2 - V^2 t(t-t_0)/t_0^2}} & \text{if} \quad 0 < t < t_0 \\ x_1^1 + \alpha \frac{p_1c^2}{E}\left(t - t_0\right) & \text{if} \quad t_0 < t \end{cases}, \qquad E = c\sqrt{m^2c^2 + p_1^2} \tag{4.12}$$

where $\alpha = \pm 1$. Sign of α and the constant x_1^1 are determined from the continuity condition of the world line at $t = t_0$. The solution (4.12) has different form, depending on the sign of the constant $4E^2 - V^2$.

If $4E^2 > V^2$, the world line (4.12) takes the form

$$x^1 = \begin{cases} x_0^1 - \frac{p_1c^2}{E} t & \text{if } t < 0 \\ x_0^1 - \frac{p_1c^2t_0}{V} \arcsin \frac{2V\left(\sqrt{E^2t_0^2 - V^2t(t-t_0)} - E(t_0 - 2t)\right)}{t_0(4E^2 + V^2)} & \text{if } 0 < t < t_0 \\ x_0^1 - p_1c^2\frac{t_0}{V} \arcsin \frac{4EV}{4E^2+V^2} - \frac{p_1c^2}{E}\left(t - t_0\right) & \text{if } t_0 < t \end{cases}, \qquad E^2 > V^2/4 \tag{4.13}$$

In the case, when $4E^2 < V^2$, the world line is reflected from the region Ω_{fb} of the space-time determined by the condition $0 < t < t_0$ in (4.11). In this case the coordinate x is not a single-valued function of the time t. We use a parametric representation for the world line (4.12). We have

$$x^1 = \begin{cases} x_0^1 - \frac{p_1c^2t_0}{2E}\left(1 - A\cosh\tau\right) & \text{if} \quad \tau < -\tau_0 \\ x_0^1 - \frac{p_1c^2t_0}{V}\left(\tau + \tau_0\right) & \text{if} \quad -\tau_0 < \tau < \tau_0 \\ x_0^1 - \frac{2p_1t_0}{V}\tau_0 + \frac{p_1c^2t_0}{2E}\left(1 - A\cosh\tau\right) & \text{if} \quad \tau_0 < \tau \end{cases} \tag{4.14}$$

$$t = \frac{t_0}{2}\left(1 - A\cosh\tau\right) \tag{4.15}$$

where

$$A = \sqrt{1 - \frac{4E^2}{V^2}}, \qquad \tau_0 = \operatorname{arccosh}\frac{1}{A} = \operatorname{arccosh}\frac{1}{\sqrt{1 - \frac{4E^2}{V^2}}} \tag{4.16}$$

The solution (4.14), (4.15) describes annihilation of particle and antiparticle with the energy $E < V/2$ in the region $0 < t < t_0$. The world line, describing the particle-antiparticle generation, has the form

$$x^1 = \begin{cases} x_0^1 - \frac{p_1 c^2 t_0}{2E}\left(A\cosh\tau - 1\right) & \text{if} \quad \tau < -\tau_0 \\ x_0^1 + \frac{p_1 c^2 t_0}{V}\left(\tau + \tau_0\right) & \text{if} \quad -\tau_0 < \tau < \tau_0 \\ x_0^1 + \frac{2p_1 t_0}{V}\tau_0 + \frac{p_1 c^2 t_0}{2E}\left(A\cosh\tau - 1\right) & \text{if} \quad \tau_0 < \tau \end{cases} \tag{4.17}$$

$$t = \frac{t_0}{2}\left(A\cosh\tau - 1\right) \tag{4.18}$$

where parameters A, τ_0 are defined by the relation (4.16), and the relation $2E < V$ takes place.

In both cases (4.14) and (4.17) at $|t| \to \infty$ the world line has two branches, which can be approximated by the relations

$$x^1 = x_0^1 + vt_1 \pm v\left(t - t_1\right), \qquad t_1 = t_0\frac{E}{V} \tag{4.19}$$

where $v = -\frac{p_1 c^2}{E}$ is the particle velocity, and $v = \frac{p_1 c^2}{E}$ is the antiparticle velocity.

The particle world line cannot turn its direction in time by means of its inner resources. It is possible only in some external field. Energy of the particle and of the antiparticle is absorbed by the external field $f(t)$. Thus, if it appears that the κ-field is not only internal field. In may be a force field which is responsible for pair production and pair annhilation, because both processes are connected with the turn of a world line in time.

5 Many stochastic relativistic particles

Let us consider N identical stochastic relativistic particles, having electrical charge e and mass m. They interact via the electromagnetic field and via the force field κ. The action has the form

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}\left[X,\kappa,A\right] = \sum_{A=1}^{A=N}\int\limits_{V_{\boldsymbol{\xi}}} L_{(A)}\left(x_{(A)}\left(\tau,\boldsymbol{\xi}\right)\right)d\tau d\boldsymbol{\xi} + \int_{V_x} L_{\mathrm{em}} d^4x \tag{5.1}$$

$$X = \left\{x_{(1)}, x_{(1)}, ...x_{(N)}\right\}, \quad x_{(A)} = \left\{x_{(A)}^0, x_{(A)}^1, x_{(A)}^2, x_{(A)}^3\right\}, \quad A = 1, 2, ...N \tag{5.2}$$

Here an index in brackets means the number of a particle.

$$L_{(A)}\left(x_{(A)}\left(\tau,\boldsymbol{\xi}\right)\right) = -mcK_{(A)}\left(x_{(A)}\right)\sqrt{g_{ik}\dot{x}_{(A)}^i\dot{x}_{(A)}^k} - \frac{e}{c}A_k\left(x_A\right)\dot{x}_{(A)}^k, \quad A = 1, 2, ...N \tag{5.3}$$

$$\dot{x}_{(A)}^i = \frac{dx_{(A)}^i}{d\tau}, \quad x_{(A)} = x_{(A)}\left(\tau,\boldsymbol{\xi}\right) \tag{5.4}$$

$$K_{(A)}=\sqrt{1+\lambda^2\left(g_{kl}\kappa^k\left(x_A\right)\kappa^l\left(x_A\right)+\frac{\partial}{\partial x^k_{(A)}}\kappa^k\left(x_A\right)\right)},\qquad \lambda=\frac{\hbar}{mc},\quad A=1,2,...N \tag{5.5}$$

$$L_{\mathrm{em}}=\frac{1}{8\pi}g^{ik}\partial_iA_l\left(x\right)\partial_kA^l\left(x\right),\quad x=\left\{x^0,x^1,x^2,x^3\right\} \tag{5.6}$$

Variation with respect to $x^i_{(A)}$ gives

$$mc\frac{d}{d\tau}\left(K_{(A)}\left(x_{(A)}\right)\frac{\dot{x}_{(A)i}}{\sqrt{\dot{x}_{(A)s}\dot{x}^s_{(A)}}}\right)-mc\frac{\partial\left(K_{(A)}\left(x_{(A)}\right)\sqrt{\dot{x}_{(A)s}\dot{x}^s_{(A)}}\right)}{\partial x^i_{(A)}}$$
$$+\frac{e}{c}\frac{d}{d\tau}A_i\left(x_A\right)-\frac{e}{c}\dot{x}^k_{(A)}\frac{\partial}{\partial x^i_{(A)}}A_k\left(x_A\right)=0,\quad A=1,2,...N \tag{5.7}$$

Variation of (5.1) with respect to $\kappa^i\left(x_{(A)}\right)$ gives

$$-mc\frac{\lambda^2g_{ki}\kappa^k\left(x_{(A)}\right)J\left(x_{(A)}\right)\sqrt{\dot{x}_{(A)s}\dot{x}^s_{(A)}}}{K_{(A)}\left(x_{(A)}\right)}+mc\frac{\partial}{\partial x^i_{(A)}}\frac{\lambda^2J\left(x_{(A)}\right)\sqrt{\dot{x}_{(A)s}\dot{x}^s_{(A)}}}{2K_{(A)}\left(x_{(A)}\right)}=0 \tag{5.8}$$

$$A=1,2,...N$$

$$J\left(x\right)=\frac{\partial\left(\tau,\xi_1,\xi_2,\xi_3\right)}{\partial\left(x^0,x^1,x^2,x^3\right)} \tag{5.9}$$

Jacobian $J\left(x_{(A)}\right)$ appears in (5.8), because one needs before variation of (5.1) to go from integration over $d\tau d\boldsymbol{\xi}$ to integration over d^4x in (5.1).

Equations (5.8) can be written in the form

$$\kappa_{(A)i}\left(x_{(A)}\right)=\frac{\partial}{\partial x^i_{(A)}}\kappa\left(x_{(A)}\right)=\frac{1}{2}\frac{\partial}{\partial x^i_{(A)}}\log\frac{J\left(x_{(A)}\right)\sqrt{\dot{x}_{(A)s}\dot{x}^s_{(A)}}}{K_{(A)}\left(x_{(A)}\right)},\quad A=1,2,...N \tag{5.10}$$

where $\kappa\left(x_{(A)}\right)$ is the potential of the κ-field κ^i.

Equations (5.10) can be integrated in the form

$$\kappa\left(x_{(A)}\right)=\frac{1}{2}\log\frac{J\left(x_{(A)}\right)\sqrt{\dot{x}_{(A)s}\dot{x}^s_{(A)}}}{K_{(A)}\left(x_{(A)}\right)}+\frac{1}{2}\log C_{(A)},\quad A=1,2,...N \tag{5.11}$$

where $C_{(A)}=C_{(A)}\left(X\right)$, $A=1,2,...N$ are functions of $X=\left\{x_{(1)},x_{(2)},..x_{(N)}\right\}$. The functions $C_{(A)}$ satisfy the conditions

$$\frac{\partial C_{(A)}\left(X\right)}{\partial x^k_{(A)}}=0,\quad A=1,2,...N,\quad k=0,1,2,3 \tag{5.12}$$

Let us note that the flux $j_{(A)}^{k}$ of Ath particle in the statistical ensemble can be presented in the form

$$j_{(A)}^{k}\left(x_{(A)}\right)=\dot{x}_{(A)}^{k}\left(\tau,\boldsymbol{\xi}\right)J\left(x_{(A)}\right) \tag{5.13}$$

and equation (5.11) can be rewritten in the form

$$\kappa\left(x_{(A)}\right)=\frac{1}{2}\log\frac{\sqrt{j_{(A)s}\left(x_{(A)}\right)j_{(A)}^{s}\left(x_{(A)}\right)}}{K_{(A)}\left(x_{(A)}\right)}+\frac{1}{2}\log C_{(A)},\quad A=1,2,...N \tag{5.14}$$

Let choose $\log C_{(A)}$ in the form

$$\log C_{(A)}=\sum_{B=1}^{B=N}\left(1-\delta_{AB}\right)\log\frac{\sqrt{j_{(B)s}\left(x_{(B)}\right)j_{(B)}^{s}\left(x_{(B)}\right)}}{K_{(B)}\left(x_{(B)}\right)} \tag{5.15}$$

According to (5.15) the κ-field at the point $x_{(A)}$ has the form

$$\kappa\left(x_{(A)}\right)=\kappa\left(x_{(A)},X_{(A)}\right)=\frac{1}{2}\sum_{B=1}^{B=N}\log\frac{\sqrt{j_{(B)s}\left(x_{(B)}\right)j_{(B)}^{s}\left(x_{(B)}\right)}}{K_{(B)}\left(x_{(B)}\right)},\quad A=1,2,...N \tag{5.16}$$

The second argument $X_{(A)}$ of κ

$$X_{(A)}=\left\{x_{(1)},x_{(2)},..x_{(A-1)},x_{(A+1)},...x_{(N)}\right\} \tag{5.17}$$

shows that the κ-field at the point $x_{(A)}$ depends on all N particles of the statistical ensemble. Using expression (5.5) for $K_{(A)}$, one can rewrite the relation (5.16) in the form of dynamic equations for $\kappa\left(x_{A}\right)$, $A=1,2,...N$.

$$\begin{aligned}&\left(1+\lambda^{2}g^{kl}\frac{\partial^{2}}{\partial x_{(A)}^{k}\partial x_{(A)}^{l}}\right)w\left(x_{A}\right)\\ =\;&\frac{j_{(A)s}\left(x_{(A)}\right)j_{(A)}^{s}\left(x_{(A)}\right)}{w^{3}\left(x_{(A)}\right)}\prod_{B=1,B\neq A}^{B=N}\frac{j_{(B)s}\left(x_{(B)}\right)j_{(B)}^{s}\left(x_{(B)}\right)w\left(x_{(B)}\right)}{K_{(B)}},\\ &A=1,2,...N\end{aligned} \tag{5.18}$$

where

$$w\left(x_{(A)}\right)=e^{\kappa\left(x_{(A)}\right)},\quad A=1,2,...N \tag{5.19}$$

Expression (5.16) is symmetric with respect to transposition of any two particles of N considered identical particles.

Although the action (5.1) is a sum of actions for single particles, and the particles look as noninteracting particles, but actually the particles interact via the κ-field. The particles interact also via electromagnetic field. The electromagnetic interaction of particles arise because of the last term in (5.1), which contains time derivatives of A_k and describes the electromagnetic field as a dynamic system. Such a term, containing time derivatives of the κ-field, is present in any $L_{(A)}$. We have seen in the fourth section that external κ-field is responsible for pair production. In this section we have seen that any relativistic particle can generate the κ-field which is external with respect to other identical particles. Any single particle generates the κ-field, which acts on the particle motion. However, at the particle description in terms of the wave function the κ-field is incorporated in the definition of the wave function by formulas (3.24). And the κ-field is considered as an attribute of the wave function describing a free quantum particle (statistical ensemble of free stochastic particles).

6 κ-field of a single particle

Let us consider an uniform statistical ensemble, whose state is described by the constant flux j^i of particles

$$j^0 = \text{const}, \quad j^\alpha = 0, \quad \alpha = 1,2,3 \tag{6.1}$$

For one particle ($N = 1$) the equation (5.14) takes the form

$$\exp(2\kappa) = \frac{\sqrt{j_s j^s}}{\sqrt{1 + \lambda^2 e^{-\kappa} g^{kl} \frac{\partial^2}{\partial x^k \partial x^l} e^\kappa}} \tag{6.2}$$

Or

$$1 + \lambda^2 e^{-\kappa} g^{kl} \frac{\partial^2}{\partial x^k \partial x^l} e^\kappa = \frac{j_s j^s}{\exp(4\kappa)} \tag{6.3}$$

Introducing designation

$$w = e^\kappa \tag{6.4}$$

one obtains dynamic equation for w

$$w + \lambda^2 \frac{\partial^2 w}{c^2 \partial t^2} - \lambda^2 \Delta w = \frac{j_s j^s}{w^3} \tag{6.5}$$

We consider the simplest case, when the flux j^k is taken in the form

$$j^\alpha = 0, \quad \alpha = 1,2,3, \quad j^0 = \begin{cases} \rho, & \text{if} \quad r < r_0 \\ 0, & \text{if} \quad r > r_0 \end{cases}, \quad \rho = \text{const} > 0, \quad r_0 \leq \lambda \tag{6.6}$$

We search for stationary spherically symmetric solution, which is an analog of the Coulomb solution for electromagnetic field. Neglecting the term with the timelike derivatives, we shall solve the equation (6.5) taken in the form

$$w - \lambda^2 \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial w}{\partial r} \right) = \frac{f}{w^3}, \quad f = j_s j^s \tag{6.7}$$

It is easy to verify that at $r < r_0$, where $f = f_0$ =const, the solution has the form $w = f_0^{1/4}$. In the region $r > r_0$, where $f = 0$ the solution has the form $w = f_0^{1/4} e^{-r/\lambda}/r$. Thus, the Coulomblike solution has the form

$$w = \begin{cases} f_0^{1/4}, \text{ if } r < r_0 \\ f_0^{1/4} \frac{e^{-r/\lambda}}{r}, \text{ if } r > r_0 \end{cases} \tag{6.8}$$

$$\kappa = \log w = -\frac{r}{\lambda} + \frac{1}{4} \log \frac{f_0}{r^4}, \quad r > r_0 \tag{6.9}$$

Of course, there is also a solution of linear equation

$$w + \lambda^2 \frac{\partial^2 w}{c^2 \partial t^2} - \Delta w = 0 \tag{6.10}$$

which takes place in the region, where $j_k = 0$.

Thus, we have investigated the case, when the external κ-field produces pairs, and the case, when the κ-field is generated by a statistical ensemble of stochastic (quantum) relativistic particles. Unfortunately, a self-consistent conception of pair production can be hardly formulated in terms of the described formalism, because this formalism does not distinguish between particles and antiparticles. In the fourth section the particle and antiparticle are distinguished by their orientation $\varepsilon = \pm 1$. But the orientation is a discrete quantity, and there is no dynamic equation for ε. We hope that one will succeed to modify the statistical ensemble formalism in such a way, that it will distinguish formally between particle and antiparticle.

7 Multivariant space-time geometry

Explanation of quantum effects by stochastic motion of elementary particles admits one to remove quantum principles as the primary laws of nature. But simultaneously the stochastic motion of free particles arises the question on reasons of the stochasticity. This stochasticity may be explained as a result of interaction with some medium (ether, vacuum) distributed in the space-time. Another reason may be an interaction of free elementary particles with the space-time directly. In other words, space-time geometry may be such one that a free elementary particle moves stochastically in this space-time geometry. World line of the stochastically moving particle wobbles. This wobbling is conditioned by a multivariance of the real space-time geometry.

Geometrical vector (g-vector) $\mathbf{AB}$ is defined as a the ordered set $\mathbf{AB} = \{A, B\}$ of two points $A, B \in \Omega$. Here Ω is the set of points (events) of the space-time, where the geometry is given. We use the term g-vector (vector), because there are linear vectors (linvectors) u, which are defined as elements of the linear vector space $\mathcal{L}_n$. Linvectors $u \in \mathcal{L}_n$ are abstract quantities, whose properties are defined by a system of axioms. In particular, operations of summation of linvectors and multiplication of a linvector by a real number are defined in $\mathcal{L}_n$. Under some conditions the operation on linvectors may be applied to g-vectors.

Linvectors and g-vectors have different properties. Any linvector exists in one copy, whereas there are many g-vectors $\mathbf{CD}$ which are equivalent to the g-vector $\mathbf{AB}$. Geometric vector $\mathbf{CD}$ is equivalent (equal) to g-vector $\mathbf{AB}$ ($\mathbf{CD}\mathrm{eqv}\mathbf{AB}$), if

$$(\mathbf{CD}\mathrm{eqv}\mathbf{AB}): \quad (\mathbf{AB}.\mathbf{CD}) = |\mathbf{CD}| \cdot |\mathbf{AB}| \wedge |\mathbf{CD}| = |\mathbf{AB}| \tag{7.1}$$

where $(\mathbf{CD}.\mathbf{AB})$ is the scalar product of two vectors $\mathbf{CD}$ and $\mathbf{AB}$, and $|\mathbf{AB}| = \sqrt{(\mathbf{AB}.\mathbf{AB})}$ is the length of the vector $\mathbf{AB}$. The two g-vectors equivalence is defined by the relation (7.1) in the proper Euclidean geometry, where

$$(\mathbf{AB}.\mathbf{CD}) = \sigma(A, D) + \sigma(B, C) - \sigma(A, C) - \sigma(B, D) \tag{7.2}$$

$$|\mathbf{AB}| = \sqrt{2\sigma(A, B)} \tag{7.3}$$

Here $\sigma(A, B)$ is the world function $\sigma(A, B) = \frac{1}{2}\rho^2(A, B)$, where $\rho(A, B)$ is the distance between the points A and B. Definition (7.1) - (7.3) of two g-vectors equivalence depends only on the world function. It does depend neither on dimension, nor on the coordinate system. Definition (7.1) - (7.3) of two g-vectors equivalence can be used in any geometry which is described completely by its world function and only by its world function. Such a geometry is called the physical geometry. If the world function is restricted by some conditions (the triangle axiom, nonnegativity of the distance ρ), such a geometry is known as metric geometry. Metric geometry is a special case of the physical geometry. The metric geometry as well as the distance geometry [11] (restricted only by the condition of nonnegativity of the distance ρ) cannot be used for description of the space-time, because in the space-time geometry the space-time distance ρ may be imaginary.

In the proper Euclidean geometry there is only one g-vector $\mathbf{CD}$ at the point C which is equivalent to the g-vector $\mathbf{AB}$ at the point A. It means that there exist only one point $D \in \Omega$ which is solution of two equations

$$(\mathbf{AB}.\mathbf{CD}) = |\mathbf{CD}| \cdot |\mathbf{AB}|, \quad |\mathbf{CD}| = |\mathbf{AB}| \tag{7.4}$$

at fixed points $A, B, C \in \Omega$.

In a physical geometry, generally speaking, there are many g-vectors $\mathbf{CD}, \mathbf{CD}'$, $\mathbf{CD}''$, ...which are equivalent to g-vector $\mathbf{AB}$. Such a geometry is considered as a multivariant geometry. The multivariance is a reason of the world line wobbling of the free particle motion. The world line is described as a set $\mathcal{C}$ of points ... $P_0, P_1, ...P_s$, ...divided by a constant distance $\rho(P_s, P_{s+1}) = \mu$, $s = ...0, 1, ...$

$$\mathcal{C} = \bigcup_s P_s, \quad \rho(P_s, P_{s+1}) = \mu = \mathrm{const}, \quad s = ...0, 1, ... \tag{7.5}$$

If the limit at $\mu \to 0$ exists, the set $\mathcal{C}$ tends to continuous world line of the particle. For free particle $(\mathbf{P}_s\mathbf{P}_{s+1}\mathrm{eqv}\mathbf{P}_{s+1}\mathbf{P}_{s+2})$, $s = ...0, 1, ...$If there is an unique solution of two equations

$$(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2}) = |\mathbf{P}_s\mathbf{P}_{s+1}| \cdot |\mathbf{P}_{s+1}\mathbf{P}_{s+2}|, \quad |\mathbf{P}_s\mathbf{P}_{s+1}| = |\mathbf{P}_{s+1}\mathbf{P}_{s+2}|, \quad s = ...0, 1, .. \tag{7.6}$$

for P_{s+2} at any given P_s, P_{s+1}, then the world line does not wobble. In this case the space-time geometry is single-variant, the limit $\mu \to 0$ exists and the point set $\mathcal{C}$ forms a continuous world line L. If the space-time geometry is multivariant, there are several point P_{s+2} determined by the points P_s, P_{s+1}. The set $\mathcal{C}$ does not form a continuous world line. The set $\mathcal{C}$ forms a wobbling broken line, consisting of connected segments of the straight line.

Even the space-time geometry of Minkowski is multivariant with respect to spacelike g-vectors. For instance, spacelike g-vectors $\mathbf{P}_{s+1}\mathbf{P}_{s+2} = \left\{\sqrt{r^2+z^2}, r\cos\phi, r\sin\phi, z\right\}$ and $\mathbf{P}_{s+1}\mathbf{P}'_{s+2} = \left\{\sqrt{r_1^2+z^2}, r_1\cos\phi_1, r_1\sin\phi_1, z\right\}$ are equivalent to the spacelike g-vector $\mathbf{P}_s\mathbf{P}_{s+1} = \{0,0,0,z\}$ at arbitrary values of quantities r, r_1, ϕ, ϕ_1. But g-vectors $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$ and $\mathbf{P}_{s+1}\mathbf{P}'_{s+2}$ are not equivalent between themselves, generally speaking. Amplitude of this difference is infinite in the sense that the value of $\left|\mathbf{P}_{s+2}\mathbf{P}'_{s+2}\right|$

$$\left|\mathbf{P}_{s+2}\mathbf{P}'_{s+2}\right|^2 = \sqrt{(r^2+z^2)\left(r_1^2+z^2\right)} - rr_1\cos(\phi-\phi_1) - z^2 \tag{7.7}$$

has neither minimum, no supremum. The particle with spacelike world line is called tachyon. Absence of supremum of (7.7) means that the world line of a tachyon wobbles with infinite amplitude, and tachyon cannot be detected, even if it exists. As far as a free tachyon cannot be detected, the contemporary scientists prefer to think that tachyons do not exist. They prefer not to consider the wobbling spacelike world lines. However, although a single tachyon cannot be detected, the tachyon gas can be detected by its gravitational field. Existence of so-called dark matter may be freely explained by a presence of the tachyon gas in cosmos [12, 13].

Tardions (i.e. particles with timelike world line) have a smooth world line in the space-time geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$, because $\mathcal{G}_{\mathrm{M}}$ is single-variant with respect to any timelike g- vectors. However, if the space-time geometry $\mathcal{G}$ differs from $\mathcal{G}_{\mathrm{M}}$, the space-time geometry $\mathcal{G}$ may be multivariant with respect to timelike g-vectors. In this case the world line of a free tardion wobbles. In particular, if the space-time geometry $\mathcal{G}_{\mathrm{d}}$ is discrete, and world function σ_{d} of this geometry $\mathcal{G}_{\mathrm{d}}$ has the form

$$\sigma_{\mathrm{d}} = \sigma_{\mathrm{M}} + \frac{\lambda_0^2}{2}\operatorname{sgn}(\sigma_{\mathrm{M}}) \tag{7.8}$$

where λ_0 is the elementary length, and σ_{M} is the world function of the geometry of Minkowski, the world lines of tardions wobble also. The discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ is given on the same manifold Ω_{M}, where the geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$ is given. But any distance ρ_{d} in the geometry $\mathcal{G}_{\mathrm{d}}$ has the property

$$|\rho_{\mathrm{d}}(P,Q)| \notin (0,\lambda_0), \quad \forall P,Q \in \Omega_{\mathrm{M}} \tag{7.9}$$

which means that any distance $|\rho_{\mathrm{d}}(P,Q)|$ is not less, than elementary length λ_0. It is easy to verify that the distance $\rho_{\mathrm{d}} = \sqrt{2\sigma_{\mathrm{d}}}$ of geometry (7.8) satisfies the condition (7.9). It follows from (7.9) that λ_0 is minimal distance in $\mathcal{G}_{\mathrm{d}}$ (but $\rho_{\mathrm{d}}(P,Q) = 0$ is possible). The discrete geometry $\mathcal{G}_{\mathrm{d}}$ is multivariant with respect to all g-vectors.

But wobbling of timelike world lines is restricted by the elementary length λ_0. This wobbling is responsible for quantum effects. If $\lambda_0^2 = \hbar/(bc)$, then statistical description of wobbling world lines is equivalent to description in terms of the Schrödinger equation [14]. Here $\hbar$ and c are respectively the quantum constant and the speed of the light. The quantity b is an universal constant which connects the geometric mass μ, defined in (7.5) with the particle mass m by means of the relation

$$m = b\mu \tag{7.10}$$

The real space-time geometry may distinguish from (7.8), but in any case the space-time geometry is multivariant, and the multivariance of the space-time geometry is a reason of quantum effects.

8 Fluidity of boundary between the particle dynamics and space-time geometry

The particle motion occurs in the space-time, and properties of the space-time are essential for description of the particle motion. The boundary between the properties of the space-time and properties of laws of motion (dynamics) is indefinite. One may choose simple properties of the space-time geometry and obtain complicated laws of dynamics. On the contrary, one may choose a simple dynamics (free particle motion) and obtain a complicated space-time geometry. It is possible intermediate version, when dynamics and space-time geometry are not very simple. Historically the boundary between physics and space-time geometry moved towards space-time geometry. This process may be qualified as the physics geometrization. One can see several steps of the physics geometrization: (1) conservation laws as a corollaries of the space-time geometry symmetry, (2) spacial relativity, (3) general relativity, (4) five-dimensional geometry of Kaluza-Klein, where motion of a charged particle in the given electromagnetic and gravitational fields is described as a free particle motion in the Kaluza-Klein space-time geometry [15].

In the classical physics, where gravitational field and electromagnetic field are the only possible force fields, the Kaluza-Klein representation realizes the complete physics geometrization. But this geometrization is not complete one in microcosm, where the quantum effects are essential. Besides, the Riemannian geometry which is used in the Kaluza-Klein description is rather complicated. The Riemannian geometry is founded on several basic concepts: (1) concepts of topology, (2) concepts of local geometry such as dimension, coordinate system, metric tensor and parallel transport. Work with concepts of the Riemannian geometry is not simpler, than the work with numerous concepts of dynamics. As a result one prefers to work with customary concepts of dynamics.

At the metric approach to geometry, when the space-time geometry is described in terms of only distance ρ or in terms of only world function $\sigma = \rho^2/2$, any modification of the space-time geometry looks very simple. To obtain a modification of a geometry, one replaces world function and obtains a modified geometry described by the new world function. If the geometry is described by means of several fundamental concepts, any modification of the geometry needs a modification of all fundamental concepts. This modification of different fundamental concepts is to be concerted, in order the modified geometry be consistent. The more number of the basic concepts the difficult agreement between the modified concepts. The monistic conception of a geometry, when there is only one fundamental quantity is the best conception, because the problem of agreement of different basic modified concepts is absent. From this viewpoint the metric approach to the space-time geometry is the best approach.

9 Metric approach to geometry and multivariance of geometry

The proper Euclidean geometry can be presented in terms and only in terms of its world function. However, attempt of generalization of the proper Euclidean geometry [11] failed in the sense, that Blumental was forced to introduce concept of continuous mapping in addition to concept of distance. The condition of the continuous mapping cannot be expressed in terms of only distance. But the continuous mapping was necessary to construct one-dimensional continuous curve in the distance geometry of Blumental. As a result Blumental failed to realize a consistent metric approach to geometry, when the geometry is discribed in terms and only in terms of a distance.

What was a reason of failure? During two thousand years we knew only proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. All statements of $\mathcal{G}_{\mathrm{E}}$ are derived logically from several basic statements (axioms). In all presentations of $\mathcal{G}_{\mathrm{E}}$ one considers the ways of derivation (theorems) of different statements of $\mathcal{G}_{\mathrm{E}}$ from axioms of $\mathcal{G}_{\mathrm{E}}$. The impression arises that these theorems form the content of $\mathcal{G}_{\mathrm{E}}$, whereas these theorems form only the way of the proper Euclidean geometry construction. The proper Euclidean geometry itself is a set $\mathcal{P}_{\mathrm{E}}$ of statements of $\mathcal{G}_{\mathrm{E}}$. At the modification $\mathcal{G}$ of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ the set $\mathcal{P}$ of statements of the geometry $\mathcal{G}$ is obtained from the set $\mathcal{P}_{\mathrm{E}}$ of statements of $\mathcal{G}_{\mathrm{E}}$. Such a derivation of $\mathcal{P}$ from $\mathcal{P}_{\mathrm{E}}$ may differ from the way of the proper Euclidean geometry construction. It is possible such a situation that the modified (generalized) geometry $\mathcal{G}$ cannot be derived from a system of axioms. In other words, the geometry $\mathcal{G}$ may be nonaxiomatizable. Unfortunately, the nonaxiomatizablity of a geometry is perceived as something impossible, and this perception is a result of identification of geometry $\mathcal{G}_{\mathrm{E}}$ with the way of derivation of $\mathcal{G}_{\mathrm{E}}$.

In general, at the metric approach to geometry the modified (generalized) geometry $\mathcal{G}$ is obtained from $\mathcal{G}_{\mathrm{E}}$ by means of a deformation, when the world function σ_{E} is replaced by the world function σ of the modified geometry $\mathcal{G}$ in all definitions and all general geometric statements containing only σ_{E}. Such a construction of a physical geometry will be referred to as the deformation principle [16, 17]. The nonaxiomatizability of a physical geometry is connected with its multivariance. Indeed, in order the logical Euclidean method of the geometry construction could work, the equivalence relation (7.1) is to be transitive, when from ($\mathbf{AB}\mathrm{eqv}\mathbf{CD}$) and ($\mathbf{AB}\mathrm{eqv}\mathbf{FH}$) follows that ($\mathbf{CD}\mathrm{eqv}\mathbf{FH}$). If the equivalence relation (7.1) is intransitive, from $(\mathbf{AB}\mathrm{eqv}\mathbf{CD}) \wedge (\mathbf{AB}\mathrm{eqv}\mathbf{FH})$ does not follows that ($\mathbf{CD}\mathrm{eqv}\mathbf{FH}$), a logical construction is impossible. But impossibility of derivation of multivariant physical geometry $\mathcal{G}$ by means of the Euclidean logical method does not mean that the set $\mathcal{P}$ of statements of a multivariant geometry $\mathcal{G}$ cannot be constructed. It can be constructed by means of the deformation principle.

Note that the Riemannian space-time geometry is multivariant with respect to remote vectors. But in the Riemannian geometry one removes fernparallelism (equivalency of remote vectors). Instead in the Riemannian geometry one introduces the parallel transport of a vector. In the Riemannian geometry the finite distance, defined as an integral along a geodesic, appears to be many-valued in many cases. Many-valued distance seems to be inadmissible from physical viewpoint.

Summation of linvectors and multiplication of a linvector by a real number are operation which are defined in the linear vector space $\mathcal{L}_n$. These operations are not adequate in application to g-vectors of multivariant geometry, although the are adequate in application to g-vectors of $\mathcal{G}_{\mathrm{E}}$, because the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ is single-variant.

Let $S_{\mathbf{AB}}$ be a set of g-vectors $\mathbf{CD}$, which are equivalent to g-vector $\mathbf{AB}$. If the equivalence relation is transitive, the set $S_{\mathbf{AB}}$ is a equivalence class $[\mathbf{AB}]$ of the g-vector $\mathbf{AB}$. It contains only g-vectors which are equivalent between themselves. In this case any equivalence class $[\mathbf{AB}]$ may be corresponded by some linvector $u \in \mathcal{L}_n$, and this correspondence will be one-to-one, because any equivalence class exist only in one copy. If the equivalence relation is intransitive and the set $S_{\mathbf{AB}}$ does not form an equivalence class, the correspondence between the linvectors and g-vectors cannot be established. As a result operation of the linear vector space $\mathcal{L}_n$ are not adequate in the multivariant geometry, where the equivalence relation is intransitive.

Formally one may introduce summation of g-vectors in multivariant geometry, but this summation will be many-valued. Let one needs to sum g-vectors $\mathbf{AB}$ and $\mathbf{CD}$, and $B \neq C$. Let g-vector $\mathbf{PQ} = \mathbf{AB} + \mathbf{CD}$, where the point P is given, and the point Q should be determined. One obtains

$$\mathbf{PQ} = \mathbf{PF} + \mathbf{FQ} \tag{9.1}$$

where points F and Q are determined from the relations

$$(\mathbf{PF}\mathrm{eqv}\mathbf{AB}) \wedge (\mathbf{FQ}\mathrm{eqv}\mathbf{CD}) \tag{9.2}$$

In the multivariant geometry the equations (9.2) has many solutions for the points F and Q, and the operation of summation appears to be many-valued. In the single-variant geometry the relations (9.2) have unique solution for points F and Q and the summation (9.1) is defined one-to-one.

Multiplication of a g-vector by a real number is also many-valued in the multivariant geometry, because definition of multiplication contains a reference to a relation of equivalence, which is many-valued in the multivariant geometry. Let $\mathbf{PQ}' = a\mathbf{PQ}$, where the points P,Q and the number a are given, then the point Q' is to be determined from the relations

$$(\mathbf{PQ}'.\mathbf{PQ}) = |\mathbf{PQ}'| \cdot |\mathbf{PQ}|, \quad |\mathbf{PQ}'| = a\,|\mathbf{PQ}| \tag{9.3}$$

Solution of the two equations is many-valued in the multivariant geometry, generally speaking. Thus, methods of differential geometry, developed for the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ are inadequate in the multivariant geometry. *However, inadequacy of the differential geometry methods in the multivariant geometry does not mean that multivariant geometries do not exist.*

The physics geometrization in the classical physics, when the space-time geometry is a Riemannian geometry, is not effective, because for determination of the Kaluza-Klein geometry one needs to determine the metric tensor g_{ik} and electromagnetic potential A_k, $k = 0,1,2,3$. However, if these quantities are known, one may write dynamic equations for the particle motion in the space-time geometry of Minkowski and determine the particle world line. A use of the Kaluza-Klein geometry appears to be needless.

In the physics geometrization inside microcosm the force fields acting on a particle are not known. They are different for different elementary particles. One supposes, that in the proper (true) space-time geometry the elementary particle motion is free. Writing dynamic equations for the free particle motion in the true space-time geometry, one may rewrite the dynamic equations in the case of the space-geometry of Minkowski. In this case the dynamic equations cease to be free dynamic equations. Dynamic equations will contain force fields, arising as a result of deflection of the Minkowski geometry $\mathcal{G}_{\mathrm{M}}$ from the true space-time geometry, where the particle motion is free. The microcosm dynamic equations in the space-time geometry of Minkowski are not known primarily. They arise as a result of transformation of free dynamic equations, written in a true space-time geometry. In the microcosm the fluidity of boundary between the particle dynamics and the space-time geometry admits one to reduce determination of the particle dynamics laws to the determination of the world function of the true space-time geometry, where the elementary particles move free.

The number of variants of the dynamics laws for indefinite number of different sorts of elementary particles is more, than the number of variants of the world functions $\sigma(P,Q)$ of two space-time points P,Q. As a result a use of the hypothesis on the boundary fluidity for any elementary particles seems to be more effective, than suppositions on dynamics of any single elementary particle, which are extracted from complicated experiments with elementary particles. Of course, the hypothesis on the boundary fluidity for any elementary particles should be tested by experiment. However, in the case of classical physics this hypothesis is true distinctly. Besides, primarily it is not clear what is responsible for peculiar properties of particle motion: the space-time geometry or the laws of the particle dynamics.

Usage of the hypothesis on the boundary fluidity between the dynamics and the space-time geometry generates a conception of the elementary particle dynamics.

In other words, a connection between the concepts of dynamics and those of the space-time geometry arise. This connection is a logical connection. It arises on the logical basis, but not on basis of a single experiment or on the basis of several single experiments. It concerns all elementary particles. This conception may appear to be valid or wrong, but it is a conception.

A like conception is absent in the contemporary elementary particle theory, where one invents suppositions on dynamics and interaction of different sorts of elementary particles, which are labelled by some quantum numbers. Absence of a conception in the contemporary theory generates numerous variants of a theory. These variants contain numerous interaction constants, which are to be determined from experiment. To understand, why it is bad, let us imagine that we have not a conception of classical particle dynamics, which states that any particle is a dynamic system, and its motion is described by a Lagrange function. In absence of such a conception one needs to invent dynamic equations for any particle, depending on its mass, color, temperature, shape and so on. In absence of the dynamics conception one cannot distinguish between essential parameters (mass) and unessential ones (color, temperature). As a result any investigation of dynamics becomes to be complicated.

10 Description of geometrical objects in multivariant geometry

A geometrical object is a geometrical image of a physical body. Any geometrical object is some subset of points in the space-time. However, a geometrical object is not an arbitrary set of points. In the physical geometry a geometrical object is to be defined in such a way, that similar geometrical objects (which are images of similar physical bodies) could be recognized in different space-time geometries.

Definition 1: A geometrical object $g_{\mathcal{P}_n,\sigma}$ of the geometry $\mathcal{G}=\{\sigma,\Omega\}$ is a subset $g_{\mathcal{P}_n,\sigma}\subset\Omega$ of the point set Ω. This geometrical object $g_{\mathcal{P}_n,\sigma}$ is a set of roots $R\in\Omega$ of the function $F_{\mathcal{P}_n,\sigma}$

$$g_{\mathcal{P}_n,\sigma}=\{R|F_{\mathcal{P}_n,\sigma}(R)=0\}, \quad F_{\mathcal{P}_n,\sigma}: \quad \Omega\rightarrow\mathbb{R} \tag{10.1}$$

where $F_{\mathcal{P}_n,\sigma}$ depends on the point R via world functions of arguments $\{\mathcal{P}_n,R\}=\{P_0,P_1,...P_n,R\}$

$$F_{\mathcal{P}_n,\sigma} \quad : \quad F_{\mathcal{P}_n,\sigma}(R)=G_{\mathcal{P}_n,\sigma}(u_1,u_2,...u_s), \qquad s=\frac{1}{2}(n+1)(n+2) \tag{10.2}$$

$$u_l = \sigma(w_i,w_k), \quad i,k=0,1,...n+1, \quad l=1,2,...\frac{1}{2}(n+1)(n+2) \tag{10.3}$$

$$w_k = P_k\in\Omega, \qquad k=0,1,...n, \qquad w_{n+1}=R\in\Omega \tag{10.4}$$

Here $\mathcal{P}_n=\{P_0,P_1,...,P_n\}\subset\Omega$ are $n+1$ points which are parameters, determining the geometrical object $g_{\mathcal{P}_n,\sigma}$

$$g_{\mathcal{P}_n,\sigma}=\{R|F_{\mathcal{P}_n,\sigma}(R)=0\}, \qquad R\in\Omega, \qquad \mathcal{P}_n\in\Omega^{n+1} \tag{10.5}$$

$F_{\mathcal{P}_n,\sigma}(R) = G_{\mathcal{P}_n,\sigma}(u_1, u_2, ...u_s)$ is an arbitrary function of $\frac{1}{2}(n+1)(n+2)$ arguments u_k and of $n+1$ parameters $\mathcal{P}_n$. The set $\mathcal{P}_n = \{P_0, P_1, ...P_n\} \in \Omega^{n+1}$ of the geometric object parameters will be referred to as the skeleton of the geometrical object. The subset $g_{\mathcal{P}_n,\sigma} \subset \Omega$ will be referred to as the envelope of the skeleton. The skeleton is an analog of a frame of reference, attached rigidly to a physical body. Tracing the skeleton motion, one can trace the motion of the physical body. When a particle is considered as a geometrical object, its motion in the space-time is described by the motion of skeleton $\mathcal{P}_n$. At such an approach (the rigid body approximation) the shape of the envelope is of no importance.

Remark: An arbitrary subset Ω' of the point set Ω is not a geometrical object, generally speaking. It is supposed, that physical bodies may have only a shape of a geometrical object, because only in this case one can identify identical physical bodies (geometrical objects) in different space-time geometries.

Existence of the same geometrical objects in different space-time regions, having different geometries, brings up the question on equivalence of geometrical objects in different space-time geometries. Such a question did not arise before, because one does not consider such a situation, when a physical body moves from one space-time region to another space-time region, having another space-time geometry. In general, mathematical technique of the conventional space-time geometry (differential geometry) is not applicable for simultaneous consideration of several different geometries of different space-time regions.

We can perceive the space-time geometry only via motion of physical bodies in the space-time, or via construction of geometrical objects corresponding to these physical bodies. As it follows from the *definition 1* of the geometrical object, the function $G_{\mathcal{P}_n,\sigma}$ as a function of its arguments u_k, $k = 1, 2, ...n(n+1)/2$ (of world functions of different points) is the same in all physical geometries. It means, that a geometrical object $\mathcal{O}_1$ in the geometry $\mathcal{G}_1 = \{\sigma_1, \Omega_1\}$ is obtained from the same geometrical object $\mathcal{O}_2$ in the geometry $\mathcal{G}_2 = \{\sigma_2, \Omega_2\}$ by means of the replacement $\sigma_2 \to \sigma_1$ in the definition of this geometrical object.

Definition 2: Geometrical object $g_{P'_n,\sigma'}$ ($\mathcal{P}'_n = \{P'_0, P'_1, ..P'_n\}$) in the geometry $\mathcal{G}' = \{\sigma', \Omega'\}$ and the geometrical object $g_{P_n,\sigma}$ ($\mathcal{P}_n = \{P_0, P_1, ..P_n\}$) in the geometry $\mathcal{G} = \{\sigma, \Omega\}$ are similar geometrical objects, if

$$\sigma'(P'_i, P'_k) = \sigma(P_i, P_k), \qquad i, k = 0, 1, ..n \tag{10.6}$$

and the functions $G'_{\mathcal{P}'_n,\sigma'}$ for $g_{P'_n,\sigma'}$ and $G_{\mathcal{P}_n,\sigma}$ for $g_{P_n,\sigma}$ in the formula (10.2) are the same functions of arguments $u_1, u_2, ...u_s$

$$G'_{\mathcal{P}'_n,\sigma'}(u_1, u_2, ...u_s) = G_{\mathcal{P}_n,\sigma}(u_1, u_2, ...u_s) \tag{10.7}$$

In this case

$$u_l \equiv \sigma(P_i, P_k) = u'_l \equiv \sigma'(P'_i, P'_k), \qquad i, k = 0, 1, ...n, \qquad l = 1, 2, ..n(n+1)/2 \tag{10.8}$$

The functions $F'_{\mathcal{P}'_n,\sigma'}$ for $g_{P'_n,\sigma'}$ and $F_{\mathcal{P}_n,\sigma}$ for $g_{P_n,\sigma}$ in the formula (10.2) have the same roots, if the relation (10.7) is fulfilled. As a result one-to-one connection between the geometrical objects $g_{P'_n,\sigma'}$ and $g_{P_n,\sigma}$ arises.

As far as the physical geometry is determined by its geometrical objects construction, a physical geometry $\mathcal{G} = \{\sigma, \Omega\}$ can be obtained from some known standard physical geometry $\mathcal{G}_{\rm st} = \{\sigma_{\rm st}, \Omega\}$ by means of a deformation of the standard geometry $\mathcal{G}_{\rm st}$. Deformation of the standard geometry $\mathcal{G}_{\rm st}$ is realized by the replacement of $\sigma_{\rm st}$ by σ in all definitions of the geometrical objects in the standard geometry. The proper Euclidean geometry $\mathcal{G}_{\rm E}$ is an axiomatizable geometry. It has been constructed by means of the Euclidean method as a logical construction. Using Euclidean method, one obtains $\mathcal{G}_{\rm E}$ in the vector representation [19]. Simultaneously the proper Euclidean geometry is a physical geometry. In this case one obtains $\mathcal{G}_{\rm E}$ in terms of the world function $\sigma_{\rm E}$, i.e. in the σ-representation [19]. It may be used as a standard geometry $\mathcal{G}_{\rm st}$. Construction of a physical geometry as a deformation of the proper Euclidean geometry $\mathcal{G}_{\rm E}$ will be referred to as the deformation principle [17]. The most physical geometries are nonaxiomatizable geometries. They can be constructed only by means of the deformation principle.

11 General geometric relations

Describing a physical geometry in terms of the world function, one should distinguish between general geometric relations and specific geometric relations. The general geometric relations are the relations, which are written only in terms of the world function. The general geometric relations are valid for any physical geometry.

The first general geometric definition is the definition of the scalar product of two vectors (7.2). Definition of the two vector equivalence (7.1) - (7.3) is also a general geometric relation.

Linear dependence of n g-vectors $\mathbf{P}_0\mathbf{P}_1, \mathbf{P}_0\mathbf{P}_2, ...\mathbf{P}_0\mathbf{P}_n$ is defined by the relation,

$$F_n(\mathcal{P}_n) = 0, \quad F_n(\mathcal{P}_n) \equiv \det ||(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)||, \qquad i,k = 1,2,...n \tag{11.1}$$

where $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$ and $F_n(\mathcal{P}_n)$ is the Gram's determinant. Vanishing of the Gram's determinant is the necessary and sufficient condition of the linear dependence of n g-vectors. Condition of linear dependence relates usually to the properties of the linear vector space. It seems rather meaningless to use it, if the linear vector space cannot be introduced. Nevertheless, the relation (11.1) written as a general geometric relation describes some general geometric properties of g-vectors, which in the proper Euclidean geometry transform to the property of linear dependence. In particular, the dimension of the proper Euclidean geometry is defined in terms of the world function by means of the relations of the type (11.1) as a maximal number of linear independent vectors, which is possible in the Euclidean space. This circumstance seems to be rather unexpected, because in conventional presentation (vector representation [19]) of the Euclidean geometry $\mathcal{G}_{\rm E}$ the geometry dimension is postulated in the beginning of the geometry construction.

The general geometric relations describe general geometric properties of g-vectors, which are used at construction of geometrical objects. General geometric relations are essentially definitions of the scalar product, equivalence of g-vectors and their linear dependence. As we have seen, a definition of geometrical objects in the form of general geometric relations (i.e. in terms of the world function) is necessary to recognize the same physical body (and corresponding geometrical object) in different space-time geometries.

The general geometric relations are parametrized by the form of the world function. Changing the form of the world function, one obtains the general geometric relations at a new value of the parameter σ (new form of the world function).

12 Specific properties of the n-dimensional Euclidean space

Along of general geometric properties, connecting mainly with the properties of the linear vector space, there are special geometric relations, describing properties of the world function. For instance, there are relations, which are necessary and sufficient conditions of the fact, that the world function σ_{E} is the world function of n-dimensional Euclidean space. They have the form [18]:

I. Definition of the dimension:

$$\exists\mathcal{P}^n \equiv \{P_0, P_1, ...P_n\} \subset \Omega, \qquad F_n\left(\mathcal{P}^n\right) \neq 0, \qquad F_k\left(\Omega^{k+1}\right) = 0, \qquad k > n \quad (12.1)$$

where $F_n\left(\mathcal{P}^n\right)$ is the n-th order Gram's determinant (11.1). Geometric vectors $\mathbf{P}_0\mathbf{P}_i$, $i = 1, 2, ...n$ are basic g-vectors of the rectilinear coordinate system K_n with the origin at the point P_0. The metric tensors $g_{ik}\left(\mathcal{P}^n\right)$, $g^{ik}\left(\mathcal{P}^n\right)$, $i, k = 1, 2, ...n$ in K_n are defined by the relations

$$\sum_{k=1}^{k=n} g^{ik}\left(\mathcal{P}^n\right) g_{lk}\left(\mathcal{P}^n\right) = \delta_l^i, \qquad g_{il}\left(\mathcal{P}^n\right) = \left(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_l\right), \qquad i, l = 1, 2, ...n \quad (12.2)$$

$$F_n\left(\mathcal{P}^n\right) = \det\left|\left|g_{ik}\left(\mathcal{P}^n\right)\right|\right| \neq 0, \qquad i, k = 1, 2, ...n \qquad (12.3)$$

II. Linear structure of the Euclidean space:

$$\sigma_{\mathrm{E}}\left(P, Q\right) = \frac{1}{2}\sum_{i,k=1}^{i,k=n} g^{ik}\left(\mathcal{P}^n\right)\left(x_i\left(P\right) - x_i\left(Q\right)\right)\left(x_k\left(P\right) - x_k\left(Q\right)\right), \qquad \forall P, Q \in \Omega \qquad (12.4)$$

where coordinates $x_i\left(P\right)$, $x_i\left(Q\right)$, $i = 1, 2, ...n$ of the points P and Q are covariant coordinates of the g-vectors $\mathbf{P}_0\mathbf{P}$, $\mathbf{P}_0\mathbf{Q}$ respectively in the coordinate system K_n. The covariant coordinates are defined by the relation

$$x_i\left(P\right) = \left(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}\right), \qquad i = 1, 2, ...n \qquad (12.5)$$

III: The metric tensor matrix $g_{lk}\left(\mathcal{P}^n\right)$ has only positive eigenvalues g_k

$$g_k > 0, \qquad k = 1, 2, ..., n \qquad (12.6)$$

IV. The continuity condition: the system of equations

$$(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}) = y_i \in \mathbb{R}, \qquad i = 1, 2, ...n \tag{12.7}$$

considered to be equations for determination of the point P as a function of coordinates $y = \{y_i\}$, $i = 1, 2, ...n$ has always one and only one solution. Conditions I – IV contain a reference to the dimension n of the Euclidean space, which is defined by the relations (12.1).

All relations I – IV are written in terms of the world function. They are constraints on the form of the world function of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. Constraints (12.1), determining the dimension via the form of the world function, look rather unexpected. They contain a lot of constraints imposed on the world function of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$, and they are necessary. At the conventional approach to geometry one uses a very simple supposition: "Let the dimension of the Euclidean space be n." instead of numerous constraints (12.1).

At the vector representation of the proper Euclidean geometry, which is based on a use of the linear vector space, the dimension is considered as a primordial property of the linear vector space and as a primordial property of the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. Situation, when the geometry dimension is different at different points of the space Ω, or when it is indefinite, is not considered. At the vector representation of the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ one does not distinguish between the general geometric relations and the specific relations of the geometry.

Instead of constraints (12.1) – (12.7) one may use an explicit form of the world function

$$\sigma_{\mathrm{E}}(x, x') = \frac{1}{2}\sum_{k=1}^{k=n}\left(x^k - x'^k\right)^2 \tag{12.8}$$

where $x^k, x'^k \in \mathbb{R}$, $k = 1, 2, ...n$ are Cartesian coordinates of points P and P' respectively. The relation (12.8) satisfies all constraints (12.1) – (12.7). It uses concepts of dimension and of coordinates as primordial concepts of geometry. Using the world function only in such an explicit form, one cannot imagine a generalized geometry without such concepts as a dimension and a coordinate system, although these concepts are only means of a geometry $\mathcal{G}_{\mathrm{E}}$ description.

In general, after the logical reloading to σ-representation, when such base concepts of $\mathcal{G}_{\mathrm{E}}$ as dimension and coordinate system are replaced by the only base concept (world function), the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ looks rather unexpected. Some concepts look very simple in the vector representation. The same concepts look complicated in the σ-representation and vice versa. As a result the proper Euclidean geometry in the σ-representation is perceived hardly. In the vector representation one has several fundamental quantities: dimension, coordinate system, linear dependence, whereas in the σ-representation there is only one fundamental quantity: world function. The dimension, the coordinate system and the linear dependence are derivative quantities. Agreement between these quantities is achieved in any physical geometry automatically, because they are defined as some attributes of a world function. But this agreement looks very strange for researchers, who learned the Euclidean geometry in its conventional presentation and believe that any properties of $\mathcal{G}_{\mathrm{E}}$ take place in any generalized geometry.

In reality $\mathcal{G}_{\mathrm{E}}$ is a degenerate geometry, where the equality relation is transitive and the property of multivariance is absent in $\mathcal{G}_{\mathrm{E}}$. According to its properties $\mathcal{G}_{\mathrm{E}}$ can be constructed as a logical construction. Most researchers believe that any space-time geometry can be derived as a logical construction. They can imagine no other method of the geometry construction. They cannot imagine that the equivalence relation may be intransitive. They assume that the equivalence relation is transitive by definition. (How can one construct a geometry, if the equivalence relation is intransitive!?). In reality such a viewpoint is a corollary of the fact that researchers have been working only with $\mathcal{G}_{\mathrm{E}}$ which is a degenerate single-variant geometry. In $\mathcal{G}_{\mathrm{E}}$ some natural geometric properties (intransitivity of the equivalence relation and multivariance) are absent. How can one accept a geometry, where customary operations: (1) summation of g-vectors, (2) multiplication of a g-vector by a number and (3) decomposition of a g-vector are inadequate?

If $\mathbf{P}_0\mathbf{P}_i$, $i = 1, 2, ...n$ are basic g-vectors in some coordinate system K_n, one can determine projections of g-vector $\mathbf{P}_0\mathbf{P}$ on the basic g-vectors

$$\mathrm{Pr}\,(\mathbf{P}_0\mathbf{P})_{\mathbf{P}_0\mathbf{P}_i} = \frac{(\mathbf{P}_0\mathbf{P}.\mathbf{P}_0\mathbf{P}_i)}{|\mathbf{P}_0\mathbf{P}_i|} \tag{12.9}$$

However, the g-vector $\mathbf{P}_0\mathbf{P}$ cannot be represented as a sum of its projections, because the summation of g-vectors is inadequate operation in the multivariant geometry. Thus, coordinates may be used for labelling of space-time points, but they cannot be used for realization of the differential geometry operations.

13 Equivalence of physical geometries

Generalization of general geometric expressions (7.1) – (7.3) on the case of the discrete geometry $\mathcal{G}_{\mathrm{d}}$ is obtained by means of the replacement of σ_{E} by σ_{d}, where σ_{d} is the world function (7.8) of the discrete geometry $\mathcal{G}_{\mathrm{d}}$. We are to be ready, that properties of concepts of dimension, linear dependence of g-vectors and segment of the straight line in $\mathcal{G}_{\mathrm{d}}$ differ strongly from their properties in $\mathcal{G}_{\mathrm{E}}$. However, we have no alternative to these relations for definition of these geometrical quantities in a discrete geometry $\mathcal{G}_{\mathrm{d}}$.

Definition 4: The physical geometry $\mathcal{G} = \{\sigma, \Omega\}$ is a point set Ω with the single-valued function σ on it

$$\sigma : \qquad \Omega \times \Omega \to \mathbb{R}, \qquad \sigma(P,P) = 0, \qquad \sigma(P,Q) = \sigma(Q,P), \qquad \forall P, Q \in \Omega \tag{13.1}$$

Definition 5: Two physical geometries $\mathcal{G}_1 = \{\sigma_1, \Omega_1\}$ and $\mathcal{G}_2 = \{\sigma_2, \Omega_2\}$ are equivalent ($\mathcal{G}_1\mathrm{eqv}\mathcal{G}_2$), if the point set $\Omega_1 \subseteq \Omega_2 \wedge \sigma_1(P,Q) = \sigma_2(P,Q)$, $\forall P, Q \in \Omega_1$, or $\Omega_2 \subseteq \Omega_1 \wedge \sigma_2(P,Q) = \sigma_1(P,Q)$, $\forall P, Q \in \Omega_2$

Remark: Coincidence of point sets Ω_1 and Ω_2 is not necessary for equivalence of geometries $\mathcal{G}_1$ and $\mathcal{G}_2$. If one demands coincidence of Ω_1 and Ω_2 in the case of equivalence of $\mathcal{G}_1$ and $\mathcal{G}_2$, then an elimination of one point P from the point set Ω_1 turns the geometry $\mathcal{G}_1 = \{\sigma_1, \Omega_1\}$ into geometry $\mathcal{G}_2 = \{\sigma_1, \Omega_1 \backslash P\}$, which appears to be not equivalent to the geometry $\mathcal{G}_1$. Such a situation seems to be inadmissible, because a geometry on a part $\omega \subset \Omega_1$ of the point set Ω_1 appears to be not equivalent to the geometry on the whole point set Ω_1.

According to definition the geometries $\mathcal{G}_1 = \{\sigma, \omega_1\}$ and $\mathcal{G}_2 = \{\sigma, \omega_2\}$ on parts $\omega_1 \subset \Omega$ and $\omega_2 \subset \Omega$ of Ω are equivalent $(\mathcal{G}_1 \mathrm{eqv} \mathcal{G}), (\mathcal{G}_2 \mathrm{eqv} \mathcal{G})$ to the geometry $\mathcal{G} = \{\sigma, \Omega\}$, whereas the geometries $\mathcal{G}_1 = \{\sigma, \omega_1\}$ and $\mathcal{G}_2 = \{\sigma, \omega_2\}$ are not equivalent, generally speaking, if $\omega_1 \nsubseteq \omega_2$ and $\omega_2 \nsubseteq \omega_1$. Thus, the relation of the geometries equivalence is intransitive, in general. The space-time geometry may vary in different regions of the space-time. It means, that a physical body, described as a geometrical object, may evolve in such a way, that it appears in regions with different space-time geometry.

The space-time geometry of Minkowski as well as the Euclidean geometry are continuous geometries. It is true for usual scales of distances. However, one cannot be sure, that the space-time geometry is continuous in microcosm. The space-time geometry may appear to be discrete in microcosm. We consider a discrete space-time geometry and discuss the corollaries of the suggested discreteness.

14 Discreteness and its manifestations

The simplest discrete space-time geometry $\mathcal{G}_\mathrm{d}$ is described by the world function (7.8). Density of points in $\mathcal{G}_\mathrm{d}$ with respect to point density in $\mathcal{G}_\mathrm{M}$ is described by the relation

$$\frac{d\sigma_\mathrm{M}}{d\sigma_\mathrm{d}} = \begin{cases} 0 & \text{if} \quad |\sigma_\mathrm{d}| < \frac{1}{2}\lambda_0^2 \\ 1 & \text{if} \quad |\sigma_\mathrm{d}| > \frac{1}{2}\lambda_0^2 \end{cases} \tag{14.1}$$

If the world function has the form

$$\sigma_\mathrm{g} = \sigma_\mathrm{M} + \frac{\lambda_0^2}{2} \begin{cases} \mathrm{sgn}\,(\sigma_\mathrm{M}) & \text{if} \quad |\sigma_\mathrm{M}| > \sigma_0 \\ \frac{\sigma_\mathrm{M}}{\sigma_0} & \text{if} \quad |\sigma_\mathrm{M}| \leq \sigma_0 \end{cases} \tag{14.2}$$

where $\sigma_0 = \mathrm{const}$, $\sigma_0 \geq 0$, the relative density of points has the form

$$\frac{d\sigma_\mathrm{M}}{d\sigma_\mathrm{g}} = \begin{cases} \frac{2\sigma_0}{2\sigma_0 + \lambda_0^2} & \text{if} \quad |\sigma_\mathrm{g}| < \sigma_0 + \frac{1}{2}\lambda_0^2 \\ 1 & \text{if} \quad |\sigma_\mathrm{g}| > \sigma_0 + \frac{1}{2}\lambda_0^2 \end{cases} \tag{14.3}$$

If the parameter $\sigma_0 \to 0$, the world function $\sigma_\mathrm{g} \to \sigma_\mathrm{d}$ and the point density (14.3) tends to the point density (14.1). The space-time geometry $\mathcal{G}_\mathrm{g}$, described by the world function (14.2) is a geometry, which is a partly discrete geometry, because it is intermediate between the discrete geometry $\mathcal{G}_\mathrm{d}$ and the continuous geometry $\mathcal{G}_\mathrm{M}$. We shall refer to the geometry $\mathcal{G}_\mathrm{g}$ as a granular geometry.

Deflection of the discrete space-time geometry from the continuous geometry of Minkowski generates special properties of the geometry, which are corollaries of impossibility of the linear vector space introduction.

Let $\mathbf{P}_0\mathbf{P}_1$ be a timelike g-vector in $\mathcal{G}_{\mathrm{d}}$ $(\sigma_{\mathrm{d}}(P_0,P_1)>0)$. We try to determine a g-vector $\mathbf{P}_1\mathbf{P}_2$ at the point P_1, which is equivalent to g-vector $\mathbf{P}_0\mathbf{P}_1$. Geometrical vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ may be considered as two adjacent links of a broken world line, describing a pointlike particle.

Let for simplicity coordinates have the form

$$P_0=\{0,0,0,0\}, \qquad P_1=\{\mu,0,0,0\}, \qquad P_2=\left\{x^0,\mathbf{x}\right\}=\left\{x^0,x^1,x^2,x^3\right\} \quad (14.4)$$

In this coordinate system the world function of geometry Minkowski has the form

$$\sigma_{\mathrm{M}}(x,x')=\frac{1}{2}\left(\left(x^0-x^{0\prime}\right)^2-(\mathbf{x}-\mathbf{x}')^2\right) \qquad (14.5)$$

and σ_{d} is determined by the relation (7.8). We are to determine coordinates x of the point P_1 from two equations (7.1), which can be written in the form

$$\sigma_{\mathrm{d}}(P_0,P_1)=\sigma_{\mathrm{d}}(P_1,P_2), \quad \sigma_{\mathrm{d}}(P_0,P_2)=4\sigma_{\mathrm{d}}(P_0,P_1) \qquad (14.6)$$

After substitution of world function (7.8) one obtains

$$\frac{1}{2}\left(\left(x^0-\mu\right)^2-\mathbf{x}^2+\lambda_0^2\right)=\frac{1}{2}\left(\mu^2+\lambda_0^2\right) \qquad (14.7)$$

$$\frac{1}{2}\left(\left(x^0\right)^2-\mathbf{x}^2+\lambda_0^2\right)=2\left(\left(x^0-\mu\right)^2-\mathbf{x}^2+\lambda_0^2\right) \qquad (14.8)$$

Solution of these equations has the form

$$x^0=2\mu+\frac{3}{2}\frac{\lambda_0^2}{\mu}, \qquad \mathbf{x}^2=3\lambda_0^2\left(1+\frac{3\lambda_0^2}{4\mu^2}\right) \qquad (14.9)$$

As a result the point P_2 has coordinates

$$P_2=\left\{2\mu+\frac{3}{2}\frac{\lambda_0^2}{\mu},r\sin\theta\cos\varphi,r\sin\theta\sin\varphi,r\cos\theta\right\}, \quad r=\lambda_0\sqrt{3+\frac{9}{4}\frac{\lambda_0^2}{\mu^2}} \qquad (14.10)$$

where θ and φ are arbitrary quantities. Thus, spatial coordinates of the point P_2 are determined to within $\sqrt{3}\lambda_0$. In the limit $\lambda_0\rightarrow 0$ the point P_2 is determined uniquely. Two solutions

$$P_2'=\left\{2\mu+\frac{3}{2}\frac{\lambda_0^2}{\mu},0,0,r\right\}, \quad P_2''=\left\{2\mu+\frac{3}{2}\frac{\lambda_0^2}{\mu},0,0,-r\right\}$$

are divided by spatial distance $i\left|\mathbf{P}_2'\mathbf{P}_2''\right|=\sqrt{4r^2+\lambda_0^2}\approx\sqrt{13}\lambda_0$ $(\lambda_0\ll\mu)$. It is a maximal distance between two solutions $\mathbf{P}_2'$ and $\mathbf{P}_2''$.

If $\lambda_0=0$, then the discrete geometry turns to the geometry of Minkowski, and $P_2=\{2\mu,0,0,0\}$. The relations

$$x^0=2\mu, \quad x^1=0, \quad x^2=0, \quad x^3=0 \qquad (14.11)$$

follow from one equation $\mathbf{x}^2 = 0$. It means, that the geometry of Minkowski is a degenerate geometry, because different solutions of the discrete geometry merge into one solution of the geometry of Minkowski.

Let us consider the same problem for spacelike g-vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$, when

$$P_0 = \{0,0,0,0\}, \quad P_1 = \{0,l,0,0\}, \quad P_2 = \{ct,x,y,z\} \tag{14.12}$$

We have the same equations (14.6), but now we have another solution

$$x = 2l + \frac{3\lambda_0^2}{2l}, \quad y = a_2, \quad z = a_3, \quad c^2t^2 = r^2 = 3\lambda_0^2 + \frac{9}{4}\frac{\lambda_0^4}{l^2} \tag{14.13}$$

where a_2 and a_3 are arbitrary numbers. The point P_2 has coordinates

$$P_2 = \left\{\sqrt{a_2^2 + a_3^2 + r^2}, 2l + \frac{3\lambda_0^2}{2l}, a_2, a_3\right\}, \quad r^2 = 3\lambda_0^2\left(1 + \frac{3\lambda_0^2}{4l^2}\right) \tag{14.14}$$

The difference between two solutions P_2' and P_2''

$$P_2' = \left\{\sqrt{a_2^2 + a_3^2 + r^2}, 2l + \frac{3\lambda_0^2}{2l}, a_2, a_3\right\}, \quad P_2'' = \left\{\sqrt{b_2^2 + b_3^2 + r^2}, 2l + \frac{3\lambda_0^2}{2l}, b_2, b_3\right\}$$

may be infinitely large

$$|\mathbf{P}_2'\mathbf{P}_2''| = \sqrt{2a_2b_2 + 2a_3b_3 - 2\sqrt{r^2 + a_2^2 + a_3^2}\sqrt{r^2 + b_2^2 + b_3^2} + 2r^2 - \lambda_0^2}$$

This difference remains very large, even if $\lambda_0 \to 0$.

Thus, both the discrete geometry and the geometry of Minkowski are multivariant with respect to spacelike g-vectors. However, this circumstance remains to be unnoticed in the conventional relativistic particle dynamics, because the spacelike g-vectors do not used there.

Multivariance of the discrete geometry leads to intransitivity of the equivalence relation of two vectors. Indeed, if ($\mathbf{Q}_0\mathbf{Q}_1\text{eqv}\mathbf{P}_0\mathbf{P}_1$) and ($\mathbf{Q}_0\mathbf{Q}_1\text{eqv}\mathbf{P}_0\mathbf{P}_1'$), but g-vector ($\mathbf{P}_0\mathbf{P}_1\overline{\text{eqv}}\mathbf{P}_0\mathbf{P}_1'$). It means intransitivity of the equivalence relation. Besides, it means that the discrete geometry is nonaxiomatizable, because in any logical construction the equivalence relation is transitive.

Transitivity of the equivalence relation in the case of the proper Euclidean geometry is a corollary of the special conditions (12.1) – (12.7). In the case of the arbitrary physical geometry they are not satisfied, in general.

Parallel transport of a g-vector $\mathbf{P}_0\mathbf{P}_1$ to some point Q_0 leads to some indeterminacy of the result of this transport, because at the point Q_0 there are many g-vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_0\mathbf{Q}_1'$,..., which are equivalent to the g-vector $\mathbf{P}_0\mathbf{P}_1$.

According to (9.1) - (9.3) results of g-vectors summation and of a multiplication of a g-vector by a real number are not unique, in general, in the discrete geometry. It means, that one cannot introduce a linear vector space in the discrete geometry.

Let the discrete geometry be described by n coordinates. Let the skeleton $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$ determine n g-vectors $\mathbf{P}_0\mathbf{P}_k$, $k = 1, 2, ...n$, which are linear independent in the sense

$$F_n(\mathcal{P}_n) = \det \|(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)\| \neq 0 \qquad i, k = 1, 2, ...n \tag{14.15}$$

One can determine uniquely projections of a g-vector $\mathbf{Q}_0\mathbf{Q}_1$ onto g-vectors $\mathbf{P}_0\mathbf{P}_k$, $k = 1, 2, ...n$ by means of relations (12.9). However, one cannot reestablish the g-vector $\mathbf{Q}_0\mathbf{Q}_1$, using its projections onto g-vectors $\mathbf{P}_0\mathbf{P}_k$, $k = 1, 2, ...n$, because a summation of the g-vector components is many-valued. Thus, all operations of the linear vector space are not unique in the discrete geometry.

Mathematical technique of differential geometry is not adequate for application in a discrete geometry, because it is too special and it is adapted for a continuous (differential) geometry. This circumstance is especially important in a description of the elementary particle dynamics. The state of a particle cannot be described by its position and its momentum, because the limit $\mu \to 0$ in (14.4) does not exist in a discrete geometry. Besides, dynamic equations cannot be differential equations.

15 Skeleton conception of particle dynamics

An elementary particle is a physical body. In the discrete space-time geometry a position of a physical body is described by its skeleton $\mathcal{P}_n = \{P_0, P_1, ..P_n\}$. Of course, such a description of a physical body position may be used in any space-time geometry. The skeleton is an analog of the frame of reference attached rigidly to the particle (physical body). Tracing the skeleton motion, one traces the physical body motion. Direction of the skeleton displacement is described by the leading vector $\mathbf{P}_0\mathbf{P}_1$.

The skeleton motion is described by a world chain $\mathcal{C}$ of connected skeletons

$$\mathcal{C} = \bigcup_{s=-\infty}^{s=+\infty} \mathcal{P}_n^{(s)} \tag{15.1}$$

Skeletons $\mathcal{P}_n^{(s)}$ of the world chain are connected in the sense, that the point P_1 of a skeleton is a point P_0 of the adjacent skeleton. It means

$$P_1^{(s)} = P_0^{(s+1)}, \qquad s = ...0, 1, ... \tag{15.2}$$

The geometric vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)} = \mathbf{P}_0^{(s)}\mathbf{P}_0^{(s+1)}$ is the leading g-vector, which determines the direction of the world chain.

If the particle motion is free, the adjacent skeletons are equivalent

$$\mathcal{P}_n^{(s)} \mathrm{eqv} \mathcal{P}_n^{(s+1)} : \qquad \mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)} \mathrm{eqv} \mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}, \qquad i, k = 0, 1, ...n, \qquad s = ..0, 1, .. \tag{15.3}$$

If the particle is described by the skeleton $\mathcal{P}_n^{(s)}$, the world chain (15.1) has $n(n+1)/2$ invariants

$$\mu_{ik} = \left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|^2 = 2\sigma\left(P_i^{(s)}, P_k^s\right), \qquad i,k = 0,1,...n, \qquad s = ...0,1,... \tag{15.4}$$

which are constant along the whole world chain.

Equations (15.3) form a system of $n\,(n+1)$ difference equations for determination of nD coordinates of n skeleton points $\{P_1, P_2, ..P_n\}$, where D is the coordinate dimension of the space-time. The number of dynamical variables, liable for determination distinguishes, generally speaking, from the number of dynamic equations. It is the main difference between the skeleton conception of particle dynamics and the conventional conception of particle dynamics, where the number of dynamic variables coincides with the number of dynamic equations.

In the case of pointlike particle, when $n = 1$, $D = 4$, the number of equations $n_e = 2$, whereas the number of variables $n_v = 4$. The number of equations is less, than the number of dynamic variables. In the discrete space-time geometry (7.8) the position of the adjacent skeleton is not uniquely determined. As a result the world chain wobbles. In the nonrelativistic approximation a statistical description of the stochastic world chains leads to the Schrödinger equations [14], if the elementary length λ_0 has the form

$$\lambda_0^2 = \frac{\hbar}{bc} \tag{15.5}$$

where $\hbar$ is the quantum constant, c is the speed of the light and b is a universal constant, connecting the particle mass m with the length μ of the world chain link by the relation (7.10).

Dynamic equations (15.3) are difference equations. At the large scale, when one may go to the limit $\lambda_0 = 0$, the dynamic equations (15.3) turn to the differential dynamic equations. In the case of pointlike particle ($n = 1$) and of the Kaluza-Klein five-dimensional space-time geometry these equations describe the motion of a charged particle in the given electromagnetic field. One can see in this example, that the space-time geometry "assimilates" the electromagnetic field. It means that one may consider only a free particle motion, keeping in mind, that the space-time geometry can "assimilate" all force fields.

Dynamic equations (15.3) realize the skeleton conception of particle dynamics in the microcosm. The skeleton conception of dynamics distinguishes from the conventional conception of particle dynamics in the relation, that the number of dynamic equations may differ from the number of dynamic variables, which are to be determined. In the conventional conception of particle dynamics the number of dynamic equations (of the first order) coincides always with the number of dynamic variables, which are to be determined. As a result the motion of a particle (or of an averaged particle) appears to be deterministic. In the case of quantum particles, whose motion is stochastic (indeterministic), the dynamic equations are written for a statistical ensemble of indeterministic particles (or for the statistically averaged particle).

In the conventional conception of the particle dynamics one can obtain dynamic equation for the statistically averaged particle (i.e. statistical ensemble normalized to one particle), but there are no dynamic equations for a single stochastic particle. In the skeleton conception of the particle dynamics there are dynamic equations for a single particle. These equations are many-valued (multivariant), but they do exist. In the conventional conception of the particle dynamics one can derive dynamic equations for the statistically averaged particle, which are a kind of equations for a fluid (continuous medium). But one cannot obtain dynamic equations for a single indeterministic particle [7].

The skeleton conception of the particle dynamics realizes a more detailed description of elementary particle. One may hope to obtain some information on the elementary particle structure.

We have now only two examples of the skeleton conception application. Considering compactification in the 5-dimensional discrete space-time geometry of Kaluza-Klein, and imposing condition of uniqueness of the world function, one obtains that the value of the electric charge of a stable elementary particle is restricted by the elementary charge [20]. This result has been known from experiments, but it could not be explained theoretically, because in the continuous space-time geometry nobody considers the world function as a fundamental quantity, and one does not demand its uniqueness.

Another example concerns structure of Dirac particles (fermions). Writing the Dirac equation as dynamic equation for an ensemble of a stochastic particle [21, 22, 23] one obtains that the mean world line of this particle is a helix with timelike axis. Spin and magnetic moment of the Dirac particle are conditioned by the particle rotation in its motion along the helical world line. Thus, statistical description provides some information on the Dirac particle structure, whereas the quantum approach cannot give such information, although in both cases one investigates the same dynamic equation.

Consideration in the framework of skeleton conception [24] shows, that a world chain of a fermion is a (spacelike or timelike) helix with timelike axis. The averaged world chain of a free fermion is a timelike straight line. The helical motion of a skeleton generates an angular moment (spin) and magnetic moment. Such a result looks rather reasonable. In the conventional conception of the particle dynamics the spin and magnetic moment of a fermion are postulated without a reference to its structure. Helical world chain of the Dirac particle is connected with the fact that the skeleton of the Dirac particle contains three, or more points and it is described by three (or more) invariants $\mu_{ik} = \mu_{ki}$, $i, k = 1, 2, 3$ which are defined by the relation (15.4). In the case of the two-point skeleton describing the pointlike particle there is only one parameter μ which describes the particle mass. At the quantum approach the parameter μ is absent and the particle mass is considered as some external (not geometric) parameter of a particle. In the skeleton conception all particle parameteres are geometric quantities. In the case of the Dirac particle its mass and spin are expressed via geometical invariants μ_{ik}. (this connection is not yet obtained).

16 Concluding remarks

Structural approach to the elementary particle theory appears as a result of a skeleton conception, where the particle state is described by means of the particle skeleton. Such a description of the particle state is relativistic. Besides, this description is coordinateless, and it can be produced in any space-time geometry (continuous or discrete). Relativistic concept of the particle state admits one to replace the quantum description by statistical description of stochastic world lines of elementary particles. As a result the quantum principles and quantum essences appeared to be unnecessary. Such a replacement of the quantum description by the statistical description appears to be possible because of a logical reloading in the particle dynamics, when a single particle as the basic object of dynamics is replaced by a statistical ensemble of particles, and dynamics of stochastic and deterministic particles is described in the same terms.

Further development of the skeleton conception arises after the logical reloading in space-time geometry, when such basic geometric concepts as dimension, coordinate system, infinitesimal distance are replaced by the unique basic concept: finite distance or world function. As a result a monistic conception of the space-time geometry appears. Capacities of this conception increases essentially because of monistic character of the description. In particular, one succeeds to overcome the degenerate character of $\mathcal{G}_{\mathrm{E}}$ and to construct multivariant space-time geometry. Multivariance of the space-time geometry explains freely the elementary particles stochasticity (quantum effects).

Thus, the supposition on the space-time geometry discreteness seems to be more natural and reasonable, than the supposition on quantum nature of physical phenomena in microcosm. Discreteness is simply a property of the space-time, whereas quantum principles assume introduction of new essences.

Formalism of the discrete geometry is very simple. It does not contain theorems with complicated proofs. Nevertheless the discrete geometry and its formalism is perceived hardly. The discrete geometry was not developed in the twentieth century, although the discrete space-time was necessary for description of physical phenomena in microcosm. It was rather probably, that the space-time is discrete in microcosm. What is a reason of the discrete geometry disregard? We try to answer this important question.

The discrete geometry was not developed, because it could be obtained only as a generalization of the proper Euclidean geometry. Almost all concepts and quantities of the proper Euclidean geometry use essentially concepts of the continuous geometry. They could not be used for construction of a discrete geometry. Only world function (or distance) does not use a reference to the geometry continuity. Only coordinateless expressions (7.1) –(7.4) of the Euclidean geometry presented in terms of world function admit one to construct a discrete geometry and other physical geometries.

Assurance, that any geometry is to be axiomatizable, was the second obstacle on the way of the discrete geometry construction. The fact, that the proper Euclidean geometry is a degenerate geometry, was the third obstacle. In particular, being a physical geometry, the proper Euclidean geometry is an axiomatizable geometry, and this circumstance is an evidence of its degeneracy. It is very difficult to obtain a general conception as a generalization of a degenerate conception, because some different quantities of the general conception coincide in the degenerate conception. It is rather difficult to disjoint them. For instance, a physical geometry is multivariant, generally speaking. Single-variant physical geometry is a degenerate geometry. In the physical geometry the straight segment $\mathcal{T}_{[P_0P_1]}$

$$\mathcal{T}_{[P_0P_1]} = \left\{ R | \sqrt{2\sigma(P_0,R)} + \sqrt{2\sigma(R,P_1)} = \sqrt{2\sigma(P_0,P_1)} \right\} \tag{16.1}$$

is a surface (tube), generally speaking. In the degenerate physical geometry (the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$) the straight segment is a one-dimensional set. How can one guess, that a straight segment is a surface, generally speaking? Besides, multivariance of the equivalence relation leads to nonaxiomatizability of geometry. But we learn only axiomatizable geometries in the last two thousand years. How can we guess, that nonaxiomatizable geometries exist? *The multivariance is a natural property of a geometry. Non-acceptance of this concept is the main reason of the discrete physical geometry disregard.* The straight way from the Euclidean geometry to physical geometries was very difficult, and the physical geometry has been derived on an oblique way.

J.L.Synge [25, 26] has introduced the world function for description of the Riemannian geometry. I was a student. I did not know the papers of Synge, and I introduced the world function for description of the Riemannian space-time in general relativity. My approach differed slightly from the approach of Synge. In particular, I had obtained an equation for the world function of Riemannian geometry [27], which contains only the world function and their derivatives,

$$\frac{\partial\sigma(x,x')}{\partial x^i} G^{ik'}(x,x') \frac{\partial\sigma(x,x')}{\partial x'^k} = 2\sigma(x,x'), \qquad G^{ik'}(x,x') G_{lk'}(x,x') = \delta_l^i, \tag{16.2}$$

where

$$G_{lk'}(x,x') \equiv \frac{\partial^2\sigma(x,x')}{\partial x^l \partial x'^k}, \quad l,k = 0,1,2,3 \tag{16.3}$$

The metric tensor is expressed via world function G by the relation

$$g_{ik}(x) = -G_{lk'}(x,x) = -\left[G_{lk'}(x,x')\right]_{x'=x} \tag{16.4}$$

but it is used at the determination of the world function $G(x,x')$ only as a initial (or boundary) condition. Equation (16.2) was obtained as a corollary of the world function definition as an integral along the geodesic, connecting points x and x'. This equation contains only world function and its derivatives, but it does not contain a metric tensor.

This equation arose the question. Let a world function G do not satisfy the equation (16.2). Does this world function describe a non-Riemannian geometry or it does describe no geometry at all? It was very difficult to answer this question. On one hand, the formalism, based on the world function, is a more developed formalism, than formalism based on a usage of metric tensor, because a geodesic between points P_0, P_1 is described in terms of the world function by algebraic equation (16.1), whereas the same geodesic is described by differential equations in terms the metric tensor.

On the other hand, the geodesic described by (16.1) is one-dimensional only in the Riemannian geometry. In n-dimensional space the equation (16.1) describes a $(n-1)$-dimensional surface. I did not know, whether the surface is a generalization of a geodesic in any geometry. I was not sure, if it is possible, because in the Euclidean geometry a straight line segment is one-dimensional by definition. I left this question unsolved and returned to it almost thirty years later, in the beginning of ninetieth.

When the string theory of elementary particles appeared, it became clear for me, that the particle may be described by means of a world surface (tube) but not only by a world line. As far as the particle world line associates with a geodesic, I decided, that a world tube may describe a particle. It meant that there exist space-time geometries, where straights (geodesics) are described by world tubes. The question on possibility of the physical space-time geometry has been solved for me finally, when the quantum description appeared to be a corollary of the space-time multivariance [14].

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